2.2 Quadratics & Conics.

Last time.

- standard form, $y = ax^2 + bx + c$
- vertex form, $y = a(x-h)^2 + k$
  - ↳ vertex is at (h,k)
- Completing the square
  - ↳ a way to get from standard to vertex form
- Drawing a parabola from an equation

set $y=0$

Finding the zeros of a quadratic.

① From vertex form

② The quadratic equation

③ Factoring (*)

we are not going to discuss factoring, however if you know how to use it feel free to.

Ex. Find the roots of $y = (x-2)^2 - 9$

$0 = (x-2)^2 - 9$

$9 = (x-2)^2$

$\sqrt{9} = x - 2$

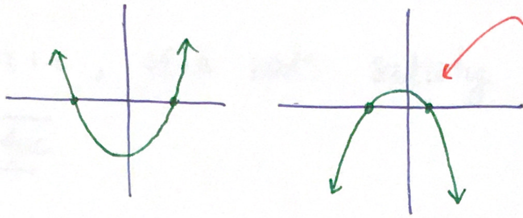
need both sq. roots of 9!

$\pm 3 = x - 2$

$x = 2 \pm 3 \Rightarrow$ zeros @ $x = 5$, $x = -1$

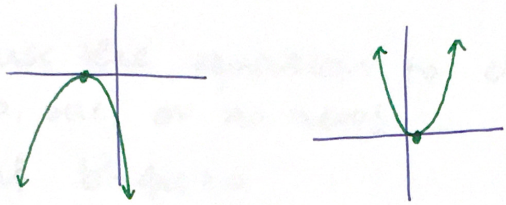
A parabola can have...

- Two roots

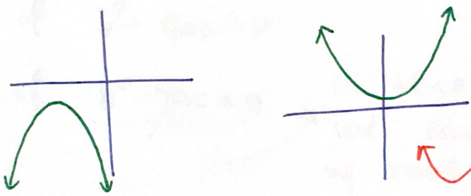


the sign of k is opposite the sign of a!

- One root



- No roots



the sign of k is the same as the sign of a!

In vertex form...

- A parabola has two roots if $\frac{k}{a} < 0$
- " ____________ " one root if $\frac{k}{a} = 0$
- " ____________ " no roots if $\frac{k}{a} > 0$

In the previous example...

$y = (x-2)^2 - 9$, $k = -9$, $a = 1$ $\Rightarrow$ $\frac{k}{a} = -9 < 0$

$\Rightarrow$ two zeros!

② Quadratic equation

Given $y = ax^2 + bx + c$, the roots satisfy

$$x_{\pm} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Again, we can use the equation to classify whether there are two, one, or no zeros:

- Two zeros if $b^2 - 4ac > 0$
- One zero if $b^2 - 4ac = 0$
- No zeros if $\underline{b^2 - 4ac < 0}$ in the above formula, we cannot take the sq. root of a negative.

<u>EX</u>. Compute zeros of $y = x^2 - 10x + 3$

$$b^2 - 4ac = (-10)^2 - 4(1)(3)$$
$$= 100 - 12 = 88 \quad \text{(two zeros!)}$$

$$x = \frac{10 \pm \sqrt{88}}{2} = \frac{10 \pm \sqrt{4 \cdot 22}}{2} = \frac{10}{2} \pm \frac{2\sqrt{22}}{2} = 5 \pm \sqrt{22}$$

Other conic sections

Shapes that occur when we slice a cone.

- Parabolas
- Circles
- Ellipses
- Hyperbolas (won't cover)

The <u>distance</u> from the point (x_1, y_1) to (x_2, y_2) is given by the expression,

$$\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$$

The order of the pts doesn't matter. This is something we will justify when we discuss right triangles.

A <u>circle</u> has the equation, $(x-h)^2+(y-k)^2=r^2$

circle is centered at (h,k)

circle has radius r

A circle is a set of points that are <u>equidistant</u> from some center.

the distance is the radius.

__Ex.__ Find the radius of the circle given by the equation, $x^2 + 2x + y^2 - 6x = 1$.

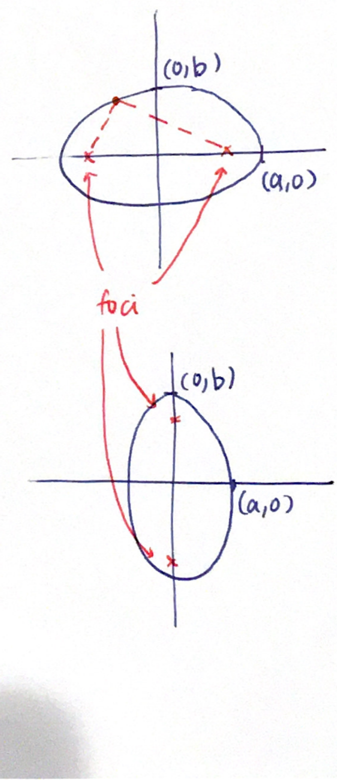
Need to __complete the square__ for x & y.

$$(x+1)^2 - 1 + (y-3)^2 - 9 = 1$$

$$(x+1)^2 + (y-3)^2 = 11 \Rightarrow r = \sqrt{11}$$

An __ellipse__ is like a stretched circle. An ellipse centered at $(0,0)$ has the equation.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



How to geometrically describe?
There are two pts called __foci__. The sum of the distances from any pt to the foci is a constant.

The axis the foci are on depend on if $a > b$ or $b > a$.

$\uparrow$ x-axis $\quad$ $\uparrow$ y-axis

<u>Foci</u> of an ellipse have the coordinates,

- $(\pm\sqrt{a^2-b^2}, 0)$ if $a>b>0$
- $(0, \pm\sqrt{b^2-a^2})$ if $b>a>0$.