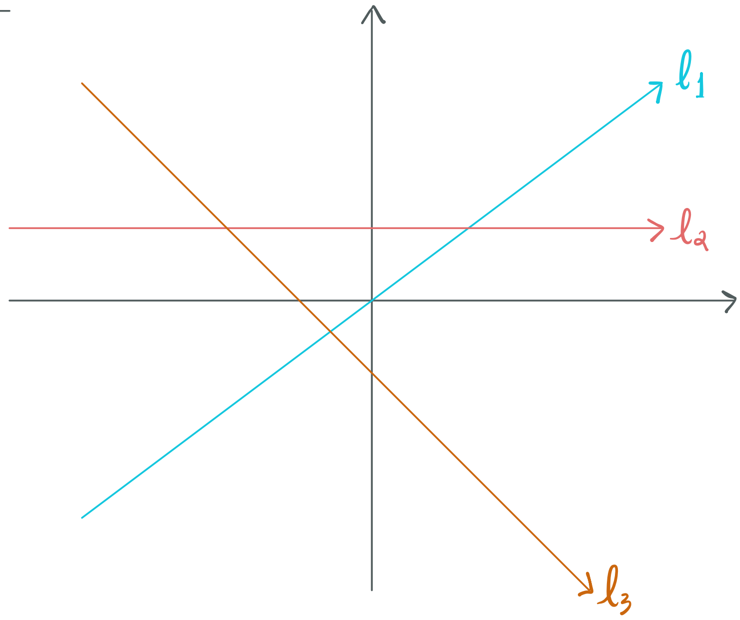
2.1 Lines.



Any line has a constant **slope**. To compute the slope of a line let (x_1, y_1) and (x_2, y_2) be any two points on the line and compute,

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

we use the variable m to denote slope

Ex. Compute the slope of the line that contains the points $(1,-1)$ and $(3,4)$.

$$m = \frac{-1-4}{1-3} = \frac{-5}{-2} = \frac{5}{2}$$

It doesn't matter which pt I chose to be (x_1, y_1)!

$$m = \frac{4-(-1)}{3-1} = \frac{5}{2}$$

Equation of a line.

(1) Pt-slope
(2) Slope-intercept

(1) Given a line with slope, m, and point, (x_1, y_1), we can write the equation,

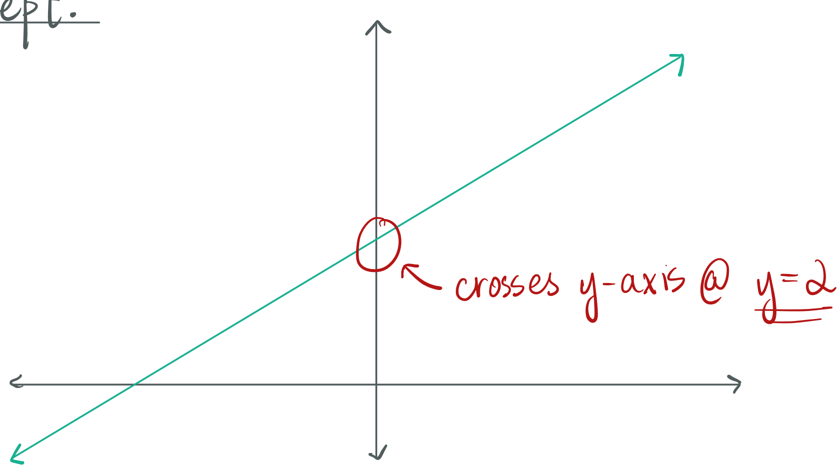
$$y - y_1 = m(x - x_1)$$

(2) Given a line with slope, m, and **y-intercept**, b, we can write the equation

$$y = mx + b$$

where the graph crosses the y-axis.

Y-intercept.



We can manipulate the equation to go from pt-slope form to slope-intercept form.

Ex. A line has pt-slope equation, $y-1=-2(x+3)$. What is the slope-intercept equation of the line.

$$y-1=-2(x+3)$$
$$y-1=-2x-6$$
$$+1 \qquad\qquad +1$$
$$y=-2x-5$$

The **constant function** (or horizontal line) is a line with slope 0.

There is a unique line that passes through any two pts.

$$\Downarrow \qquad\qquad \Downarrow \qquad\qquad \Downarrow$$

Given two pts we can write the equation of a line.

Ex. Compute the pt-slope form of the line that passes through $(2,1)$ and $(3,0)$.

$$\frac{1-0}{2-3} = \frac{1}{-1} = -1 = m$$

$y - 0 = -1(x-3)$ OR $y-1 = -1(x-2)$

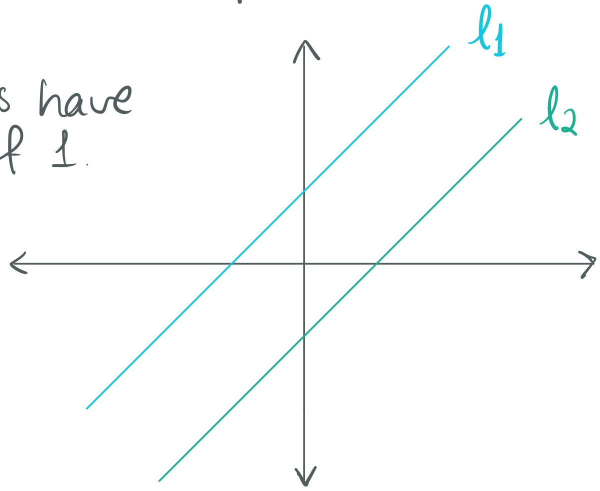
let's show that these are equivalent by simplifying to slope-intercept form

$$y = -1(x-3)$$
$$y = -x + 3$$

$$y-1 = -1(x-2)$$
$$y-1 = -x+2$$
$$y = -x+3$$

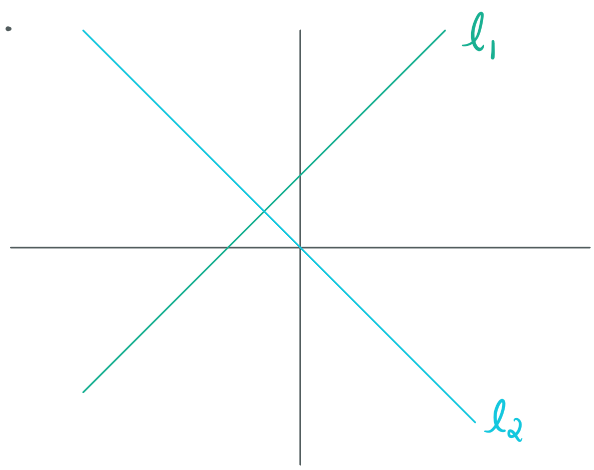
Two lines are **parallel** if and only if they have the same slope.

Both lines have a slope of 1.



Fact. Parallel lines only intersect if they are the same line.

Perpendicular lines are lines that intersect at a 90° angle.



Two non-vertical lines are perpendicular if and only if $\underbrace{\text{the product of their slopes is } -1.}_{}$

in other words, the slopes are **opposite reciprocals**

Ex. Give an example of a line that is perpendicular to $y = 3x + 1$.

The slope of our new line needs to be $-\frac{1}{3}$.
Any line with this slope will work.
For example, $y = -\frac{1}{3}x + 2$.

Ex. Find a line that is perpendicular to $y = -2x + 3$ and passes through the point $(1, 2)$.

$m = \frac{1}{2}$ (in order to be $\perp$)

$y = \frac{1}{2}x + b$ (use the pt to solve for b)

$2 = \frac{1}{2}(1) + b$

$2 = \frac{1}{2} + b$

$\frac{3}{2} = b$ $\Rightarrow$ $y = \frac{1}{2}x + \frac{3}{2}$

Exercise. Find a line that is parallel to $y = 4x - 1$ that passes through the point $(-3, 1)$.

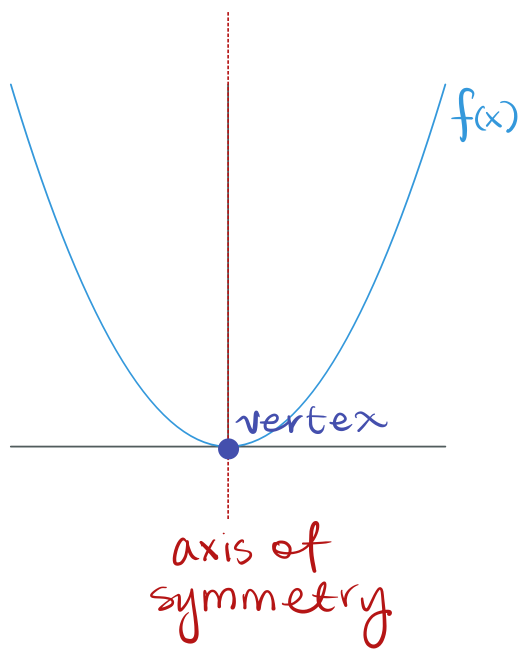
2.2 Quadratic functions & Conics.

A **quadratic function** has the form(s),

① $y = ax^2 + bx + c$ (standard form)
② $y = a(x-h)^2 + k$ (vertex form)

with $a \neq 0$. We call the shape of a quadratic a **parabola**.

Ex. $f(x) = x^2$



Vertex form, $y = a(x-h)^2 + k$, is named because the vertex of the parabola is at (h, k).

Ex. Going from standard → vertex form, AKA **completing the square**.

$y = x^2 + 4x - 1$

$y = (x^2 + 4x) - 1$

$y = (x^2 + 4x + (\frac{4}{2})^2) - 1 - (\frac{4}{2})^2$

$y = (x^2 + 4x + 2^2) - 1 - 4$

$y = (x+2)^2 - 5$

When we complete the square we use the formula,

$x^2 + bx = (x + \frac{b}{2})^2 - (\frac{b}{2})^2$

Ex. Write the following parabola in vertex form,
$y = x^2 - 2x + 3.$

$y = (x^2 - 2x + 1) + 3 - 1$

$y = (x-1)^2 + 2$

Things we like to know about quadratics:

(1) Does it face up or down?

(2) Where is its vertex?

(3) Does it have roots/zeros and what are they? (*) discussed more in 2.4

(1) If $a > 0$, the parabola faces up. If $a < 0$, the parabola faces down.