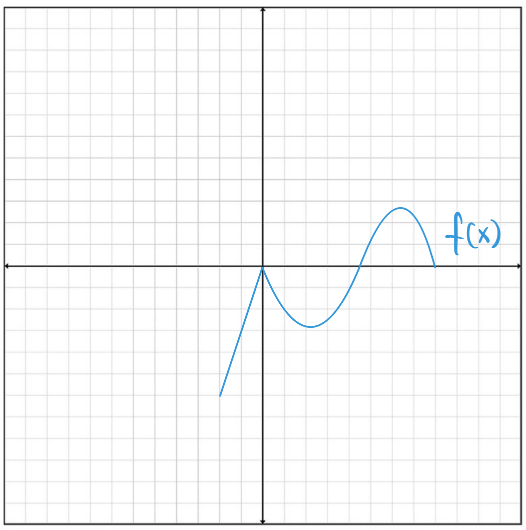
(*)

Exercise. Describe the order of transformations for $g(x) = \frac{1}{3} f(-x+1) - 3$.

Exercise Draw $g(x) = 2f(x-3) - 1$ for f below,



Even & odd functions.

A function is **even** if $f(-x) = f(x)$.

A function is **odd** if $f(-x) = -f(x)$.

Warning: functions can be neither!

Ex. is $f(x) = x^2$ even, odd, or neither?

$f(-x) = (-x)^2 = x^2 = f(x)$

$f(x)$ is even.

Exercise. is $f(x) = x^3 + x$ even, odd, or neither?

1.4 Composition of functions.

We can easily define normal operations on products.

$$(f+g)(x) = f(x) + g(x)$$
$$(f-g)(x) = f(x) - g(x)$$
$$(fg)(x) = f(x) \cdot g(x)$$
$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$$

Ex. $f(x) = x^2$, $g(x) = \sqrt{x-1}$, $h(x) = \sqrt{10-x}$

$$(fg)(x) = x^2\sqrt{x-1}$$
$$(g+h)(x) = \sqrt{x-1} + \sqrt{10-x}$$
$$\left(\frac{f}{h}\right)(x) = \frac{x^2}{\sqrt{10-x}}$$

Exercise. What are the domains of the above functions?

The **composition** of f and g, denoted $f \circ g$ is the function defined by

read "f of g"

$$(f \circ g)(x) = f(g(x))$$

<u>Ex.</u> $f(x) = x+2$, $g(x) = \frac{1}{x^2}$. Compute $f \circ g$ and $g \circ f$

$$(f \circ g)(x) = f(g(x))$$
$$= f\left(\frac{1}{x^2}\right) = \frac{1}{x^2} + 2$$

$$(g \circ f)(x) = g(f(x)) = g(x+2)$$
$$= \frac{1}{(x+2)^2}$$

<u>Domain.</u> the domain of $f \circ g$ is the set of numbers x in the domain of g such that $g(x)$ is in the domain of f.

<u>Ex.</u> $f(x) = \frac{1}{x}$, $g(x) = \sqrt{8-x}$

What is the domain of $f \circ g$?

$$(f \circ g)(x) = \frac{1}{\sqrt{8-x}}$$

Domain is all x such that $8 - x > 0$

$$-x > -8$$

$$x < 8$$

Exercise. What is the domain of $g \circ f$ for f and g given above?

The **identity function** is the function, $I(x) = x$ for all x.

For any function f, $f \circ I = I \circ f = f$.

Ex. Let $F(x) = (x-1)^2$. Write F as the composition of two functions

(*) there is no systematic way of accomplishing this, just make your best guess and check

$f(x) = x^2$

$g(x) = x - 1$

$(f \circ g)(x) = (x-1)^2$ ✓

(*) There can be many answers!

$f(x) = x+1$ $\qquad$ $(f\circ g)(x) = x^2 - 2x + 1$

$g(x) = x^2 - 2x$ $\qquad$ $= (x-1)^2$

<u>Exercise.</u> Let $F(x) = \sqrt{\dfrac{x-1}{x+3}}$. Find two functions such that F is equal to their composition.

Composition is **associative** meaning,

$$(f\circ g)\circ h = f\circ (g\circ h)$$

for any functions $f, g,$ and h.

<u>Ex.</u> Let $g(x) = x^2$, $f(x) = x+1$, $h(x) = \sqrt{x+1}$.
Compute $h\circ f\circ g$.

$$h\circ f\circ g = h(f(g(x))) = h(f(x^2))$$

$$= h(x^2+1) = \sqrt{x^2+2}$$

<u>Exercise.</u> Compute $g\circ f\circ h$.

A **linear function** is a function of the form, $h(x) = mx + b$, where $m, b \in \mathbb{R}$.

Functional transformations can be expressed as compositions with linear functions.

<u>Ex.</u> $g(x) = 2f(x) - 1$

Let $h(x) = 2x - 1$, then $g(x) = (h \circ f)(x)$.

<u>Exercise</u> CHECK!

<u>Ex.</u> $g(x) = f(-x + 1)$

Let $h(x) = -x + 1$, $g(x) = (f \circ h)(x)$.

Vertical transformations: compose w/ a linear function on the <u>left</u>.

Horizontal transformations: compose w/ a linear function on the <u>right</u>.

Ex. $g(x) = -f(x-2)+5$

$h_1(x) = x-2$

$h_2(x) = -x+5$

$g(x) = (h_2 \circ f \circ h_1)(x)$

Exercise. Let $g(x) = 2f(2x+1)-3$. Write g as a composition of f and linear functions.