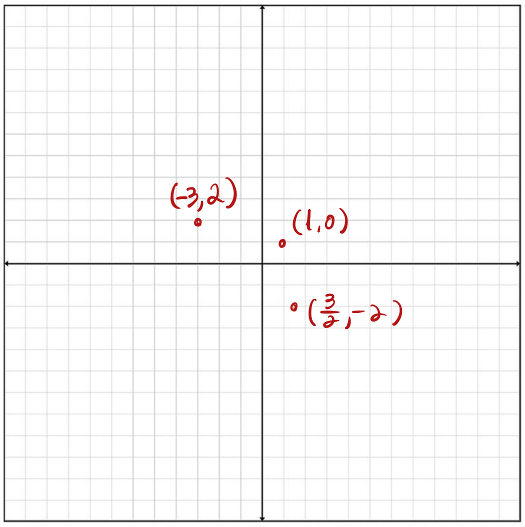
<u>Warm up.</u> Plot the pts $(1,0)$, $(-3,2)$, and $(\frac{3}{2},-2)$ on the coord grid.



From looking at the graph of a function we can:

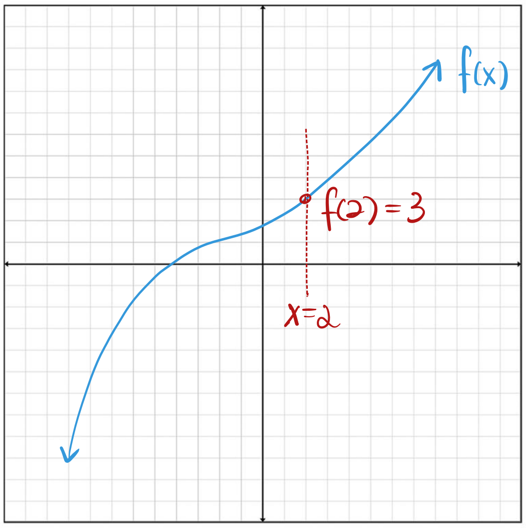
(1) estimate the value of the function at a point,

(2) determine for what input a function attains a certain output,

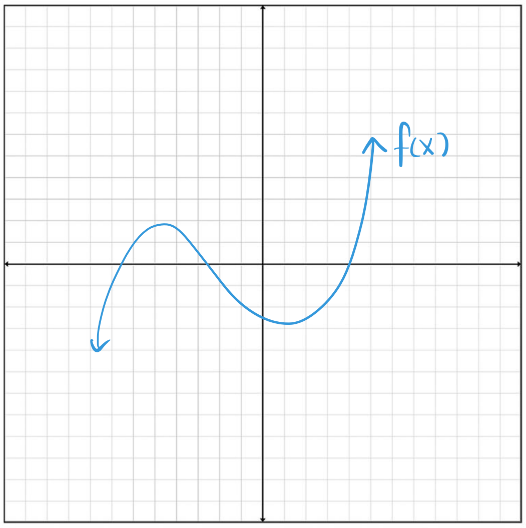
(3) estimate the range of a function.

(1) To estimate $f(a)$, for $a \in \mathbb{R}$, draw the vertical line $x=a$ and find where it intersects the graph of $f(x)$.

Ex. Estimate $f(2)$ for the function below,

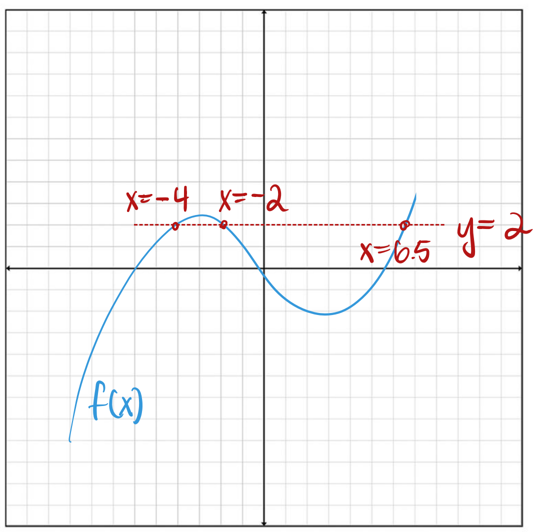


Exercise. Estimate $f(-1)$.



(2) To estimate x such that $f(x) = a$, find where the graph of f intersects the horizontal line $y = a$.

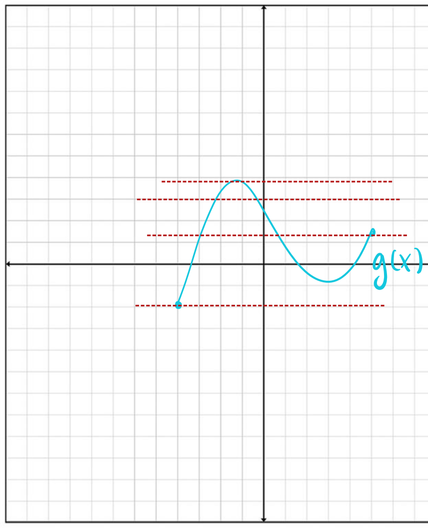
<u>Ex.</u> When does $f(x) = -2$ for the function f below?



<u>Note</u>: there can be multiple values of x that satisfy $f(x) = a$!

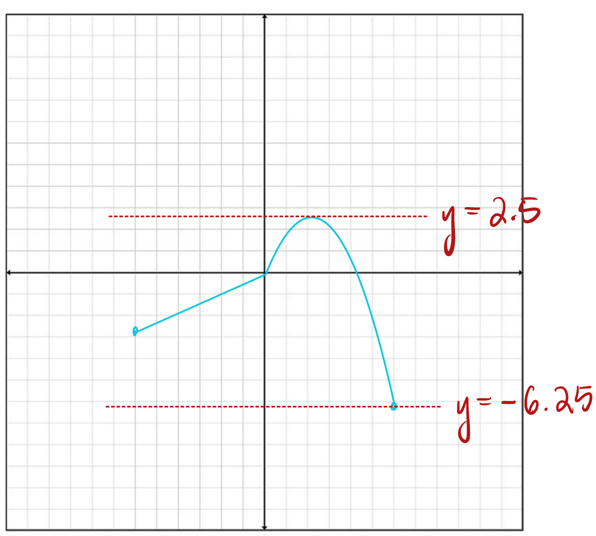
(3) A number c is in the range of f if and only if the horizontal line $y = c$ intersects the graph of f.

The function g with domain $[-4, 5]$ is shown below,



The range of this function is $[-2, 3.9]$

<u>Ex.</u> What is the range of the function below, which has domain $[-6, 6]$.



1.3 Functional transformations

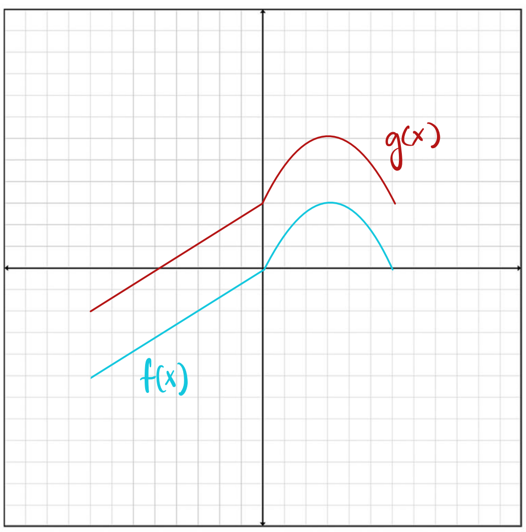
(1) $f(x)$ - function

$a > 0$

$g(x) = f(x) + a$ shift the graph of f up a units

$h(x) = f(x) - a$ shift down a units

Ex. Given $f(x)$ below, draw $g(x) = f(x) + 3$

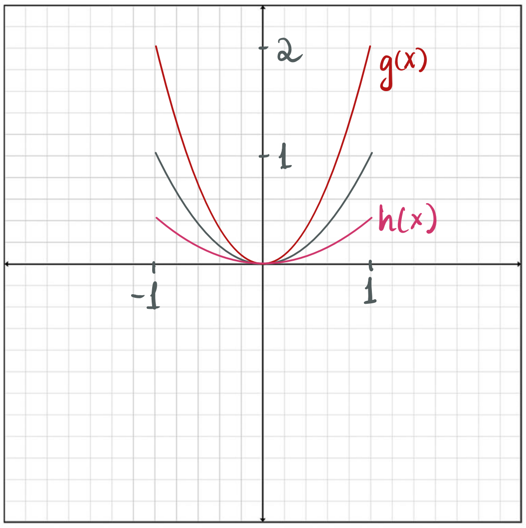


(2) $f(x)$ - function

$c > 0$

$g(x) = cf(x)$ vertically stretch the graph by c.

Ex. Consider $f(x) = x^2$ on the domain $[-1,1]$

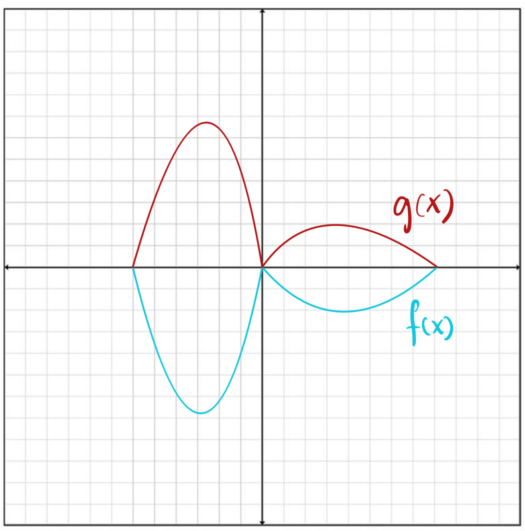


Draw $g(x) = 2x^2$ and $h(x) = \frac{1}{2}x^2$

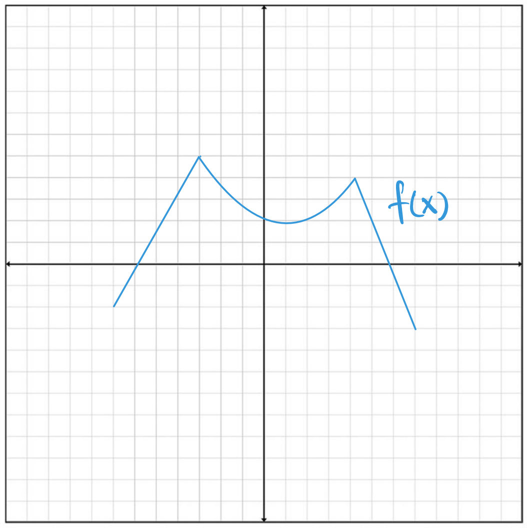
Exercise. What are the ranges of the functions above?

(3) $g(x) = -f(x)$ flip the graph of f across the horizontal axis.

Ex. Let $g(x) = -f(x)$ where $f(x)$ is shown below. Draw g.



Exercise. Given f below, draw $g(x) = -f(x)$.



(4) $b > 0$

$g(x) = f(x+b)$ shift f left b units to get g

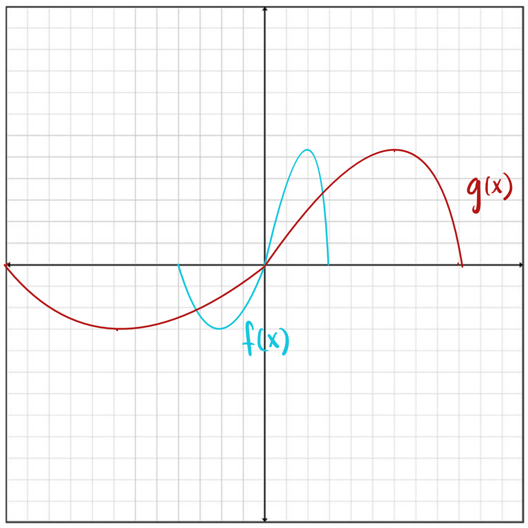
$h(x) = f(x-b)$ shift f right b units to get h

(5) $c > 0$

$g(x) = f(cx)$ stretch the graph of f horizontally by a factor of $\frac{1}{c}$ to obtain g.

(6) $g(x) = f(-x)$ flip the graph of f across the vertical axis to get g

Ex. Draw $g(x) = f\left(\frac{x}{3}\right)$ ← stretch by 3



Combining transformations.

Apply the transformations in the order that corresponds to order of operations when evaluating functions.

Ex. $g(x) = -2f(x-1)+1$

① shift right
② stretch vertically
③ flip about horizontal axis
④ shift up.