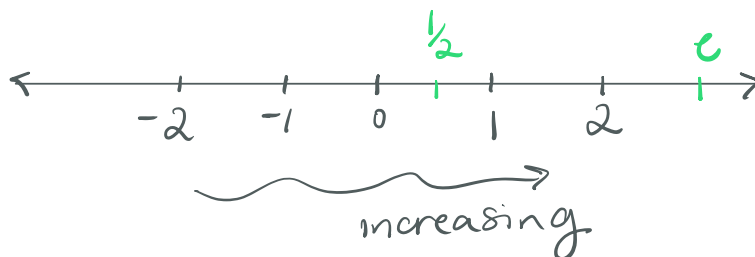


0.3 Inequalities & intervals.

number line.



Given real numbers a, b exactly one of the following is true:

(1) $a < b$

(2) $a = b$

(3) $a > b$

$<$ less than

$>$ greater than

$=$ equal to

\leq less than or equal to

\geq greater than or equal to

Ex. $1 = 1$

$\pi > e$

$-3 \leq -2$

$-4 < \frac{5}{7}$

Properties:

(1) Transitive, $a < b$ and $b < c \Rightarrow a < c$
 $5 \leq 7$ and $7 \leq 9 \Rightarrow 5 \leq 9$

(2) Addition, $a < b$ and $c < d \Rightarrow a + c < b + d$
 $1 < 3$ $-1 < 5 \Rightarrow 0 < 8$

"implies"
↳ "then"

(3) Multiplication, $a < b$ and $c > 0 \Rightarrow ac < bc$

$a < b$ and $c < 0 \Rightarrow ac > bc$

(4) Additive inverses, $a < b \Rightarrow -a > -b$

just there above $c = -1$

Ex $\frac{x-1}{x-3} < 2$ *assumed $x-3 > 0$.*

$x-1 < 2(x-3)$ *mult $x-3$.*

$x-1 < 2x-6$ *-x both sides*

$-1 < x-6$ *+6 both sides*

$5 < x$ *$x > 3$*

$x > 5$ ✓

What about $x=0$?

$\frac{0-1}{0-3} = \frac{1}{3} < 2$??

Now assume $x-3 < 0 \Rightarrow x < 3$:

$x-1 > 2(x-3)$

$x-1 > 2x-6$

$5 > x$ *$x < 3$*

$x < 5$

Solution $x > 5$ and $x < 3$

Ex. $\frac{x+2}{x-1} < 3$

$x-1 > 0$:

$(x-1) \frac{x+2}{x-1} < 3(x-1)$

$x+2 < 3x-3$

$5 < 2x \Rightarrow x > \frac{5}{2}$

(*) $x-1 < 0$: $(x-1) (*)$

$(x-1) \frac{x+2}{x-1} > 3(x-1)$

$x+2 > 3x-3$

$5 > 2x$

$x < \frac{5}{2}$

