Inner product, length, and orthogonality **Name:**

(1) Let $\boldsymbol{u}, \boldsymbol{v} \in \mathbb{R}^n$.

(a) $\boldsymbol{u} \cdot \boldsymbol{v} =$

(b) List the four properties of the inner product:

(i)

(ii)

(iii)

(iv)

(c) $\|\boldsymbol{v}\| =$

(2) Let $\boldsymbol{u} = \begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix}$, $\boldsymbol{v} = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$, and $\boldsymbol{w} = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$ and compute the following

(a) $(\boldsymbol{u} \cdot \boldsymbol{v})\boldsymbol{w}$

(b) $(\boldsymbol{v} \cdot \boldsymbol{w})\boldsymbol{u}$

(c) $\text{dist}(\boldsymbol{u}, \boldsymbol{v}) - \text{dist}(\boldsymbol{v}, \boldsymbol{w})$

(d) $\text{dist}(\boldsymbol{u}, \boldsymbol{w})$

(3) Find a nonzero vector $\boldsymbol{v}$ that is orthogonal to $\boldsymbol{u_1} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$ and $\boldsymbol{u_2} = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$.

(4) Find a matrix A with dimensions 3×3 such that the columns are orthogonal but the rows are not.

(5) Determine whether the following are true or false. Justify your answer.

(a) Let $\{\boldsymbol{v}_1, \boldsymbol{v}_2\}$ be an orthogonal set, then $\{c_1\boldsymbol{v}_1, c_2\boldsymbol{v}_2\}$ is an orthogonal set for any c_1, c_2.

(b) If $\boldsymbol{u} \perp \boldsymbol{v}$ and $\boldsymbol{v} \perp \boldsymbol{w}$, then $\boldsymbol{u} \perp \boldsymbol{w}$.

(c) If $\boldsymbol{u} \perp \boldsymbol{v}$ and $\boldsymbol{v} \perp \boldsymbol{w}$, then $\text{dist}(\boldsymbol{u}, \boldsymbol{w}) = \text{dist}(\boldsymbol{u}, \boldsymbol{v}) + \text{dist}(\boldsymbol{v}, \boldsymbol{w})$.

(d) Let $\boldsymbol{a}_1, \ldots, \boldsymbol{a}_n$ be the columns of the matrix A and suppose $\boldsymbol{a}_i \perp \boldsymbol{a}_j$ for $i \neq j$, then $AA^T = D$, where D is diagonal.

(e) Let A be a square matrix with orthogonal columns $a_1, \ldots, a_n$ and $\|a_i\| \neq 0$ for all $i = 1, \ldots, n$, then A is invertible.