(1) List the 8 axioms of a **vector space**:

(a)
(b)
(c)
(d)
(e)
(f)
(g)
(h)

(2) List the three properties of a **subspace**:

(a)
(b)
(c)

(3) The following are NOT vector spaces. Name an axiom (there may be more than one) they fail to satisfy:

(a) Let V be the set of points in the upper half plane of of $\mathbb{R}^2$, with usual addition and scalar multiplication.
(b) Let $V = \{(x, y, z) \mid x = y + 2,\ z = 3y\}$ with usual addition and scalar multiplication.
(c) As a set let $V = \mathbb{R}^2$. Define scalar multiplication as usual, but define addition by,

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \oplus \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} x_1 + y_2 \\ x_2 + y_1 \end{bmatrix}$$

(4) Show that $P_2(\mathbb{R}) = \{ax^2 + bx + c \mid a, b, c \in \mathbb{R}\}$ is a vector space by either showing that it satisfies the axioms of a vector space or by showing is it a subspace of $C(\mathbb{R})$.

(5) Let W be the set of all vectors of the form $\begin{bmatrix} a+2b \\ 3b \\ a \end{bmatrix}$. Find $\boldsymbol{u}$ and $\boldsymbol{v}$ such that $W = \text{Span}\{\boldsymbol{u}, \boldsymbol{v}\}$. For what n is W a subspace of $\mathbb{R}^n$?

(6) *Mathematically* define the **kernel** and **range** of a linear transformation (how Paulin defines them in his notes is "mathematical").

(7) Consider the transformation $T : P_2(\mathbb{R}) \to \mathbb{R}^2$ defined by, $p(x) \mapsto (p'(1), p'(0))$ where $p(x)$ is some element of $P_2(\mathbb{R})$ (which we defined in 3).

(a) Show that T is a linear transformation.

(b) Compute the kernel and range of T.

(c) Show that the kernel and range of T are subspaces of $P_2(\mathbb{R})$ and $\mathbb{R}^2$, respectively.