Rank and Nullity **Name:**

(1) Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation. Define the following,

(a) rank T

(b) the rank theorem (sometimes called the rank-nullity theorem)

(2) Let T be a linear transformation represented by a matrix A and suppose T' is the linear transformation represented by the matrix A^T, where $(A^T)_{ij} = A_{ji}$.

(a) Let A be an $n \times m$ matrix. What is the domain and codomain of T and T'?

(b) Prove that the $\dim \operatorname{col} A^T = \dim \operatorname{row} A$.

(c) Prove that $\operatorname{rank} T = \operatorname{rank} T'$. Hint: use the REF of A.

(d) When is T' the inverse of T? Why is this not always the case?

(3) Let A be an $m \times n$ matrix that represents a linear transformation, $T : \mathbb{R}^n \to \mathbb{R}^m$. Restate the following properties in terms of: rank A, col A, row A, and null A (you don't have to use all of them).

(a) T is onto.

(b) T is one-to-one.

(c) A is invertible.

(d) T is the zero transformation.

(4) Answer the following true or false questions. Justify your answers.

(a) If A is a 6×7 matrix and $RREF(A)$ has 6 pivotsm, then the map given by multiplication by A is one-to-one.

(b) If A is a 4×5 matrix and B is a 5×3 matrix then rank $A \leq$ rank B.

(c) If A is an $n \times n$ rank matrix such that rank $(A^2) < n$, then rank $A < n$ as well.

(d) If A is a 4×5 matrix and B is a 5×3 matrix, then rank $(AB) \leq$ rank A.