

Lecture 1-22

Saturday, January 20, 2018 10:46 AM

Important topics:

- Echelon form & reduced echelon form
- Pivot & free columns

Let's put some matrices in reduced row echelon form,

Ex.
$$\begin{bmatrix} 1 & -2 & 3 \\ 2 & 0 & 1 \\ -1 & 3 & 4 \end{bmatrix}$$

add 1st row to 3rd row.
$$\begin{bmatrix} 1 & -2 & 3 \\ 2 & 0 & 1 \\ 0 & 1 & 7 \end{bmatrix}$$

add (-2) × 1st row to 2nd row.
$$\begin{bmatrix} 1 & -2 & 3 \\ 0 & 4 & -5 \\ 0 & 1 & 7 \end{bmatrix}$$

mult. 2nd row by $\frac{1}{4}$
$$\begin{bmatrix} 1 & -2 & 3 \\ 0 & 1 & -\frac{5}{4} \\ 0 & 1 & 7 \end{bmatrix}$$

3rd row - 2nd row
$$\begin{bmatrix} 1 & -2 & 3 \\ 0 & 1 & -\frac{5}{4} \\ 0 & 0 & \frac{33}{4} \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 3 \\ 0 & 1 & -\frac{5}{4} \\ 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

check that we've satisfied the properties of RREF.

Ex. $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 2 & -1 & 2 & 0 \\ 0 & 3 & 2 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & -1 & 2 & -2 \\ 0 & 3 & 2 & -1 \end{bmatrix}$

$\rightarrow \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & -2 & 2 \\ 0 & 3 & 2 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & -2 & 2 \\ 0 & 0 & 8 & -7 \end{bmatrix}$

$\rightarrow \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & -2 & 2 \\ 0 & 0 & 1 & -7/8 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1/4 \\ 0 & 0 & 1 & -7/8 \end{bmatrix}$

Annotations:
 - Red arrows point to the circled '2' and '3' in the first two matrices, labeled "pivot".
 - A red circle highlights the last column of the final matrix, with an arrow pointing to it and the text "this column is free."
 - Below the final matrix, it says "check that this is RREF."

SUMMARY: pivot columns should have one 1 and zeros elsewhere, free columns are a linear combination of the columns before them.

Ex. Consider the augmented matrix,

$$\left[\begin{array}{ccc|c} 1 & -3 & 4 & 0 \\ 2 & -6 & 3 & 1 \end{array} \right]$$

1. Write out the linear system this describes.

$$x - 3y + 4z = 0$$

$$2x - 6y + 3z = 1$$

2. Write the left side of the augmented matrix in reduced echelon form & solve the system.

$$\begin{bmatrix} 1 & -3 & 4 & | & 0 \\ 0 & 0 & -5 & | & 1 \end{bmatrix} \xrightarrow{\text{next pivot}} \begin{bmatrix} 1 & -3 & 4 & | & 0 \\ 0 & 0 & 1 & | & 1/5 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 1 & -3 & 0 & -4/5 \\ 0 & 0 & 1 & 1/5 \end{bmatrix}$$

y is free
 $x = -4/5 + 3y$ SOL'N
 $z = 1/5$