Lecture 1-19

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Systems of Linear Equations

3 Possible Sol'n Types:

- 1 sol'n CONSISTENT & UNIQUE
- no sol'n
- ∞-many sol'n CONSISTENT

Ex 1.

$$\left.\begin{array}{l} x+3y=4 \\ 2x-y=1 \end{array}\right\} \Rightarrow \begin{array}{l} x+3y=4 \\ \underline{6x-3y=3} \\ 7x \quad =7 \\ \boxed{x=1} \end{array}$$

one sol'n

$$1+3y=4 \Rightarrow 3y=3 \Rightarrow \boxed{y=1}$$

Geometrically: two lines (in 2D space) that intersect at the point (1,1).

Ex 2.

$$\begin{array}{l} x+2y-z=0 \\ \underline{\quad 2y+z=1} \\ x+4y \quad\; =1 \Rightarrow x=1-4y \end{array}$$

$$(1-4y)+2y-z=0$$

$$1=2y+z$$

$$\boxed{y=\tfrac{1}{2}-\tfrac{1}{2}z}$$

$$x=1-(2-2z)$$

$$\boxed{x=-1+2z}$$

Have ∞-many sol'n because z can be any thing.

Geometrically: intersection of two planes at a line (the specific line is best describe parametrically by the above, where z is the parameter).

You probably learned this method a while ago, and know that it gets tedious pretty fast. So lets use augmented matrices and elementary row operations to solve larger systems.

Ex 3.

$$\left.\begin{aligned} 2x + y - 3z &= 0 \\ x - y + z &= 1 \\ 3y + 2z &= 3 \end{aligned}\right\} \Rightarrow \left[\begin{array}{ccc|c} 2 & 1 & -3 & 0 \\ 1 & -1 & 1 & 1 \\ 0 & 3 & 2 & 3 \end{array}\right]$$

Would like to make this at least upper triangular (or better yet the identity matrix).

switched rows 1 & 2 (optional)

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 2 & 1 & -3 & 0 \\ 0 & 3 & 2 & 3 \end{array}\right]$$

when I refer to row 1, I mean the row 1 of the prior step

add $-2\times(\text{row}1)$ to row 2

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 0 & 3 & -5 & -2 \\ 0 & 3 & 2 & 3 \end{array}\right]$$

add $-1\times(\text{row}2)$ to row 3

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 1 \\ 0 & 3 & -5 & -2 \\ 0 & 0 & 7 & 5 \end{array}\right]$$

The augmented matrix is now upper triangular. I could keep using ERO, but I have done enough work to solve the system.

$x - y + z = 1$ ①

$3y - 5z = -2$ ②

$7z = 5$ ③

solve ③, then ②, then ①

$\boxed{z = \frac{5}{7}}$

$3y - \frac{25}{7} = -2 \Rightarrow 3y = -\frac{14}{7} + \frac{25}{7} = \frac{11}{7}$

$\boxed{y = \frac{11}{21}}$

$x - \frac{11}{21} + \frac{15}{21} = \frac{21}{21}$

$\boxed{x = \frac{17}{21}}$

<u>Exercises</u>. Solve the following using the augmented matrix & ERO technique.

1. $x + 2y - z = 4$

   $x - y = 2$

   $2x - y + 2z = -8$

   SOL'N: $\boxed{x = \frac{6}{7},\ y = \frac{-8}{7},\ z = \frac{-38}{7}}$

2. $x + 3z = 4$

   $2x + 2y - 4z = 0$

   $x + z = 4$

   SOL'N: $\boxed{x = 4,\ y = -4,\ z = 0}$

3. $x + 2y - z + \omega = 0$

   $2x + 3z - \omega = 1$

   $y + 4z + 2\omega = 4$

   $x + y + z = 1$

   SOL'N: $\boxed{x = -1 \quad y = \frac{10}{9} \quad z = \frac{8}{9} \quad \omega = -\frac{1}{3}}$