Inverses and Determinants of Matrices **Name:**

(1) Determine whether or not the following matrices are invertible. If they are, compute their inverse.

(a) $\begin{pmatrix} 2 & 1 \\ 0 & -4 \end{pmatrix}$

(b) $\begin{pmatrix} 3 & 7 & -2 \\ 2 & 0 & 1 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & 0 & 0 \\ 2 & 3 & 3 \\ 0 & 1 & 1 \end{pmatrix}$

(d) $\begin{pmatrix} 0 & 1 & 0 \\ 2 & 2 & 2 \\ -1 & 3 & 1 \end{pmatrix}$

(2) Compute the determinant of the following square matrices.

(a) $\begin{pmatrix} 3 & 2 \\ 1 & -4 \end{pmatrix}$

(b) $\begin{pmatrix} 2 & 1 & 0 \\ 3 & 4 & 1 \\ 0 & 0 & 5 \end{pmatrix}$

(c) $\begin{pmatrix} 1 & 2 & 7 & 8 \\ 0 & -2 & 1 & 9 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & -2 \end{pmatrix}$

(d) $\begin{pmatrix} -1 & 4 & 2 \\ 0 & 3 & 1 \\ 2 & 4 & 5 \end{pmatrix}$

(3) Prove or disprove the following statements about inverses:

(a) If A and B are invertible $n \times n$ matrices, then AB is invertible.

(b) Let $T(\boldsymbol{x}) = A\boldsymbol{x}$, if A is invertible as a matrix then the transformation T is onto and one-to-one.

(c) If A and B are invertible $n \times n$ matrices, then $A + B$ is invertible.

(4) Prove or disprove the following statements about determinants:

(a) Assume A is invertible, then $\det(A^{-1}) = \det(A)^{-1}$

(b) $\det(A + B) = \det(A) + \det(B)$

(c) If A and B are $n \times n$ matrices such that the columns of B are linearly dependent, then $\det(AB) = 0$.

(d) If A and B are $n \times n$ matrices such that the columns of B are linearly dependent, then $\det(A + B) = 0$.