Lecture 3-5

Monday, March 5, 2018 8:58 AM

Eigenvalues and eigenvectors:

Def. An eigenvalue, λ, and eigenvector, $\underline{x}$, satisfy the equations $\underline{x} \neq 0$ and

$$A\underline{x} = \lambda \underline{x}$$

A is square or else this equation doesn't make sense.

If you want to think of A as a linear trans. A preserves the direction of $\underline{x}$, but not necessarily the magnitude.

Compute eigenvalues: solve $\det(A - \lambda I) = 0$.

sometimes you might see $\lambda I - A$, there's no diff.

(1) $A = \begin{bmatrix} 4 & 1 \\ 2 & 0 \end{bmatrix}$

$$A - \lambda I = \begin{bmatrix} 4-\lambda & 1 \\ 2 & -\lambda \end{bmatrix}$$

$$\det(A - \lambda I) = (4-\lambda)(-\lambda) - 2$$

$$= -4\lambda + \lambda^2 - 2 \overset{?}{=} 0$$

$$\lambda^2 - 4\lambda - 2 \overset{?}{=} 0$$

$$\lambda = \frac{4 \pm \sqrt{16+8}}{2} = 2 \pm \sqrt{6}$$

(2) $A = \begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix}$

$$A - \lambda I = \begin{bmatrix} 1-\lambda & 1 \\ 3 & 4-\lambda \end{bmatrix}$$

this is called the characteristic polynomial

$$\det(A - \lambda I) = (1-\lambda)(4-\lambda) - 3$$

$$= 4 - 4\lambda - \lambda + \lambda^2 - 3$$

$$= \lambda^2 - 5\lambda + 1 \stackrel{?}{=} 0$$

$$\lambda = \frac{5 \pm \sqrt{25-4}}{2}$$

$$= \frac{5}{2} \pm \frac{1}{2}\sqrt{21}$$

(3) $A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$

$$A - \lambda I = \begin{bmatrix} 1-\lambda & 2 \\ 2 & 4-\lambda \end{bmatrix}$$

$$\det(A - \lambda I) = (1-\lambda)(4-\lambda) - 4$$

$$= 4 - 5\lambda + \lambda^2 - 4$$

$$= \lambda^2 - 5\lambda \stackrel{?}{=} 0$$

$$\lambda = 0, 5$$

this is because A is not invertible... why?

(4) $A = \begin{bmatrix} 2 & 3 & 1 \\ 0 & -2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Since A is upper triangular,

$\lambda = \pm 2, 1.$

(5) $A = \begin{bmatrix} 0 & 0 & 0 \\ 2 & 3 & -1 \\ 1 & 0 & 4 \end{bmatrix}$

$$A - \lambda I = \begin{bmatrix} -\lambda & 0 & 0 \\ 2 & 3-\lambda & -1 \\ 1 & 0 & 4-\lambda \end{bmatrix}$$ cofactor expand

$$\det(A - \lambda I) = -\lambda \begin{vmatrix} 3-\lambda & -1 \\ 0 & 4-\lambda \end{vmatrix}$$

$$= -\lambda(3-\lambda)(4-\lambda) = 0$$

$$\lambda = 0, 3, 4.$$

again because A is not invertible.

Now we want to find the eigenvectors corresponding to a given eigenvalue.

Given λ, $\underline{x}$ that satisfies $A\underline{x} = \lambda \underline{x}$ is $\underline{x}$ s.t. $\underline{x} \in \text{Null}(A - \lambda I)$. ($\underline{x} \neq \underline{0}$!)

The eigenspace of an eigenvalue λ is all possible eigenvectors of λ and $\underline{0}$.

(6) Compute the eigenvectors for (3) above.

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$$

$\lambda = 0$: $A - \lambda I = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$

$$\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \underline{0}$$ $\Leftarrow$ and thus in the nullspace

$$\left.\begin{aligned} x_1 + 2x_2 &= 0 \\ 2x_1 + 4x_2 &= 0 \end{aligned}\right\} \text{same equation}$$

$x_1 = -2x_2$

standard to choose a value for the free variable.

$$\underline{x} = \begin{bmatrix} -2x_2 \\ x_2 \end{bmatrix} \longrightarrow \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

Picking a basis elt of the eigenspace.