Math 54 Sp2020 - 3/10/20

Determinants **Name:**

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(1) Compute the determinant of the following matrices.

(a) $\begin{bmatrix} 1 & -2 \\ 0 & 6 \end{bmatrix}$ $\det\left(\begin{bmatrix} 1 & -2 \\ 0 & 6 \end{bmatrix}\right) = 1\cdot 6 - 0(-2)$

$= 6$

(b) $\begin{bmatrix} 2 & -3 \\ 1 & 5 \end{bmatrix}$ $\det\left(\begin{bmatrix} 2 & -3 \\ 1 & 5 \end{bmatrix}\right) = 2\cdot 5 - 1(-3)$

$= 10 + 3 = 13$

cofactor expand

(c) $\begin{bmatrix} 0 & 1 & -1 \\ 2 & 0 & 1 \\ -1 & 3 & 1 \end{bmatrix}$ $\det([\cdots]) = -1\cdot \begin{vmatrix} 2 & 1 \\ -1 & 1 \end{vmatrix} + (-1)\cdot \begin{vmatrix} 2 & 0 \\ -1 & 3 \end{vmatrix}$

$= -1(2+1) - 1(6+0)$

$= -3 - 6 = -9$

(d) $\begin{bmatrix} 1 & 3 & 10 & -2 \\ 0 & 1 & 4 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ $\det([\cdots]) =$ product of diagonals.

$= 1\cdot 1\cdot 1\cdot 1 = 1.$

Upper Δ

cofactor expand

(e) $\begin{bmatrix} 1 & 0 & 1 & -4 \\ 0 & 2 & 4 & 0 \\ 0 & 0 & 1 & -1 \\ -1 & 3 & 0 & 2 \end{bmatrix}$ $\det([\cdots]) = 1\cdot \begin{vmatrix} 2 & 4 & 0 \\ 0 & 1 & -1 \\ 3 & 0 & 2 \end{vmatrix} - (-1)\begin{vmatrix} 0 & 1 & -4 \\ 2 & 4 & 0 \\ 0 & 1 & -1 \end{vmatrix}$

$= 2\begin{vmatrix} 1 & -1 \\ 0 & 2 \end{vmatrix} - 4\begin{vmatrix} 0 & -1 \\ 3 & 2 \end{vmatrix} - 2\begin{vmatrix} 1 & -4 \\ 1 & -1 \end{vmatrix}$

$= 2(2) - 4(3) - 2(-1+4) = -14$

(2) Determine whether the following statements are true or false. If false, give a counter example. If true, prove why.

(a) Let A be an $n \times n$ matrix. If the columns of A are linearly dependent, then $\det A = 0$.

True. col A LD $\Rightarrow$ A is not invertible

$$\Rightarrow \det(A) = 0$$

$\square$

(b) Suppose A is an upper triangular $n \times n$ matrix, then A is invertible.

False. $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ $\det(A) = 0 \cdot 0 - 1 \cdot 0 = 0$

if A is upper triangular, then $\det(A) = 0$ if one (or more) diagonal terms are 0.

(c) Let A be an $n \times n$ matrix, then $\det A = -\det(-A)$.

False. $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $-A = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

we can see that $\det(A) = \det(-A) = 1$.

(d) Let A and B be $n \times n$ matrices. If AB is invertible then both B and A are invertible.

True. AB invertible $\Rightarrow \det(AB) \neq 0$

$$\Rightarrow \det(A)\det(B) \neq 0$$

$\det(AB) = \det(A)\det(B)$

$$\Rightarrow \det(A) \neq 0 \text{ AND } \det(B) \neq 0 \Rightarrow$$ A & B are invertible.

(3) Show that if A is invertible, then $\det(A^{-1}) = \dfrac{1}{\det A}$.

By the definition of the inverse,

$$AA^{-1} = I$$

Apply the det to both sides,

$$\det(AA^{-1}) = \det(I)$$

$\det(AB) = \det(A)\det(B)$

$$\det(A)\det(A^{-1}) = 1$$

$$\det(A^{-1}) = 1/\det(A) \qquad \square$$

(4) Find a formula for $\det(rA)$ in terms of r and $\det A$, where r is a constnant and A is an $n \times n$ matrix

rA multiplies each row of A by r. There are n rows, so by theorem 3 (section 3.2) we have

$$\det(rA) = r^n \det(A).$$

$=$

A' but all rows are multiplied by r.