Determine whether the following statements are true or false...

1. Let A be an invertible $n \times n$ real matrix, then the rows of A span $\mathbb{R}^n$.

2. Let A be invertible, then A is diagonalizable.

3. Let M and N be matrices such that the product MN exists. Then $\operatorname{rank}(MN) = \operatorname{rank}(N)$.

4. Let A and B be $n \times n$ matrices. If AB is invertible, then A and B are invertible.

5. The space of solutions of the differential equation $y''(t) + y(t) = e^t$ has dimension 3.

6 Consider the transformation $T : \mathbb{P}_2 \to \mathbb{P}_2$ such that $T(p(x)) = p'(x) + p(x)$.

6a Show that T is a linear transformation.

6b Compute the matrix $[T]_{\mathcal{E}}$ where $\mathcal{E}$ is the standard basis of $\mathbb{P}_2$, $\{1, x, x^2\}$.

6c Does there exist a transformation $S : \mathbb{P}_2 \to \mathbb{P}_2$ such that $S(T(p(x))) = p(x)$ and $T(S(p(x))) = p(x)$ for all $p(x) \in \mathbb{P}_2$.

6d Does there exist a basis $\mathcal{B}$ of $\mathbb{P}_2$ such that $[T]_{\mathcal{B}}$ is diagonal? If so, compute $\mathcal{B}$.

7 Solve the initial value problem,

$$y''(t) + 5y'(t) - 6y(t) = t \quad , \qquad y(0) = 1 \quad y'(0) = 0 .$$

Determine whether the following statements are true or false...

8 For any real matrix A, the matrix A^TA is diagonalizable.

9 If A is an $n \times n$ matrix such that $\text{rank}(A^2) < n$, then $\text{rank}(A) < n$

10 Let A be a skew-symmetric $(A = -A^T)$, real matrix, then A is invertible.

11 If A is symmetric then A^{-1} is symmetric.

12 Let $y_1(t)$ and $y_2(t)$ be solutions to $y''(t) - y(t) = te^{2t}$. If $y_1(t_0) = y_2(t_0)$ and y_1'(t_0) = y_2'(t_0) for some $t_0 \in \mathbb{R}$, then $y_1(t) = y_2(t)$ for all t.