Lec 24 Worksheet

For questions 1 - 8 compute the power series for the function. Assume the center of the series is 0, unless otherwise indicated.

1. $f(x) = \dfrac{5}{1 - 4x^2}$

2. $f(x) = \dfrac{x^2}{x^4 + 16}$

3. $f(x) = x \tan^{-1}(x^3)$

4. $f(x) = \displaystyle\int_1^x \frac{\tan^{-1} t}{t}\, dt$

5. $f(x) = \displaystyle\int_0^x t^2 \ln(1 + t)\, dt$

6. $f(x) = \dfrac{2x + 3}{x^2 + 3x + 2}$

7. $f(x) = \dfrac{x - 1}{x + 2}$, $a = 1$ (try $a = 0$ too)

8. $f(x) = \dfrac{5(x - 3)}{4 - x}$, $a = 3$

9. Show that

$$J_0(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{2^{2n}(n!)^2}$$

Satisfies the differential equation

$$x^2 J_0''(x) + x J_0'(x) + x^2 J_0(x) = 0$$

10. By completing the square show that

$$\int_0^{1/2} \frac{dx}{x^2 - x + 1} = \frac{\pi}{3\sqrt{3}}$$

Now by factoring $x^3 + 1$ as a sum of cubes, rewrite the integral above. Then express $1/(x^3 + 1)$ as the sum of a power series and use it to prove the following formula,

$$\pi = \frac{3\sqrt{3}}{4} \sum_{n=0}^{\infty} \frac{(-1)^n}{8^n} \left(\frac{2}{3n + 1} + \frac{1}{3n + 2} \right)$$