

Compute the following limits.

1.
$$\lim_{n \rightarrow \infty} \frac{\sqrt{n^3 + n - \frac{1}{n}}}{n + 3}$$

2.
$$\lim_{n \rightarrow \infty} n \sin\left(\frac{1}{n^2}\right)$$

3.
$$\lim_{n \rightarrow \infty} \frac{\cos^2 n}{\ln n}$$

4.
$$\lim_{n \rightarrow \infty} (-1)^n \sqrt{\frac{n+3}{n-6}}$$

5.
$$\lim_{n \rightarrow \infty} \sqrt[n]{\sin^2 n}$$

For the following prove whether the series converges absolutely, converge conditionally, or diverges.

1.
$$\sum_{k=1}^{\infty} \sqrt{k} \left(1 - \cos\left(\frac{1}{k}\right)\right)$$

2.
$$\sum_{k=1}^{\infty} \frac{\sqrt{(2k)!}}{k!}$$

3.
$$\sum_{k=1}^{\infty} (-1)^k (k+1)^{1/k}$$

4.
$$\sum_{k=1}^{\infty} (-1)^k \sin\left(\frac{1}{\sqrt{k}}\right)$$

5.
$$\sum_{k=1}^{\infty} \frac{3^{3k}}{k\sqrt{k}}$$

6.
$$\sum_{k=1}^{\infty} \frac{(-1)^k}{\ln(k!)}$$

Determine whether the series converges or diverges, if the series is convergent, determine it's sum.

1.
$$\sum_{k=1}^{\infty} \frac{2k+3}{k^2-1}$$

2.
$$\sum_{k=1}^{\infty} \frac{2}{k^2+3k+2}$$

3.
$$\sum_{k=1}^{\infty} \ln\left(\frac{k+1}{k+2}\right)$$

4.
$$\sum_{k=1}^{\infty} \frac{e^k + 2^{2k+1}}{3^{2k}}$$

5. $\sum_{k=1}^{\infty} \frac{k^2}{2^k}$

Determine whether the following statements are true or false. If false provide a counter example.

1. The limit of the sequence $\{a_n\}$ is 0 and the sequence $\{b_n\}$ diverges, therefore the sequence $\{a_n b_n\}$ diverges.
2. The sequence $\{a_n\}$ diverges and the series $\sum b_n$ converges, then the series $\sum a_n b_n$ diverges.
3. If $\lim_{n \rightarrow \infty} |a_n| = 0$ then the series $\sum a_n$ converges.
4. If $\sum a_n$ diverges, then $\sum |a_n|$ diverges.
5. Every bounded sequence converges.
6. Every bounded convergence sequence is monotone.
7. If the sequence $\{a_n\}$ diverges and the sequence $\{b_n\}$ diverges, then the sequence $\{a_n b_n\}$ also diverges.

Find the interval and radius of convergence of the following power series.

1. $\sum_{n=0}^{\infty} \frac{(-5)^n}{n^2 3^{2n}} (x-7)^n$

2. $\sum_{n=0}^{\infty} \frac{n^n x^{2n}}{n^3}$

3. $\sum_{n=0}^{\infty} n! (x-3)^n$

4. $\sum_{n=0}^{\infty} \frac{(n^2)!}{n^{n^2} (x-1)^n}$

5. $\sum_{n=0}^{\infty} \frac{(-1)^n (5x-7)^n}{3^n (n^2+4)}$

Compute the Taylor series of the following functions,

1. $f(x) = \cosh x$

2. $f(x) = \left(\frac{x}{1-x}\right)^6$

3. $f(x) = x \cos(x^2/2)$

4. $f(x) = (x)^{1/3}, a = 8$

5. $f(x) = x^2 \ln(1+x^3)$

Compute the first four non-zero terms of the Taylor series for the following functions

1. $f(x) = e^{\sin x}$

2. $f(x) = (\sin x)^2$

3. $f(x) = \frac{xe^{3x}}{x^2 + 1}$

4. $f(x) = \sec^2 x$

5. $f(x) = \sqrt{x + 1} \cos(x)$