Exam 1 Review (sections 7.1 - 7.5, 7.8)

Recommended Book Exercises: (from Chapter 7 Review, starting page 537) 6, 8, 10, 12, 16, 18, 24, 28, 36, 37, 40, 42, 47, 48, 71, 78, 79.

And the following problems...

Compute the following integrals,

1. $\displaystyle\int \frac{x^4}{(1+x^5)^2}\,dx$
2. $\displaystyle\int x^4(\ln x)^3\,dx$
3. $\displaystyle\int_0^1 \frac{e^{2x}}{1+e^x}\,dx$
4. $\displaystyle\int \cos^4 x \sin^2 x\,dx$
5. $\displaystyle\int \tan^3 x\,dx$
6. $\displaystyle\int \frac{x^3+x^2}{x^2(x+2)}\,dx$
7. $\displaystyle\int \frac{dx}{x^2+5x+1}$
8. $\displaystyle\int \frac{x^4}{x^4-1}\,dx$
9. $\displaystyle\int \frac{2x^2-x+20}{(x-2)(x^2+9)}\,dx$
10. $\displaystyle\int \frac{\sqrt{x^2+1}-\sqrt{x^2-1}}{\sqrt{x^4-1}}\,dx$
11. $\displaystyle\int \frac{\sec^2 x}{9+\tan^2 x}\,dx$
12. $\displaystyle\int \frac{dx}{\sqrt{x-1}+\sqrt{(x+1)^3}}$
13. $\displaystyle\int \frac{\arctan(x)}{x^2+1}\,dx$
14. $\displaystyle\int \frac{2x\sin(x)}{\cos^3(x)}\,dx$

15. $\displaystyle\int \frac{\cos x - \sin x}{\sin x + \cos x}\, dx$

16. $\displaystyle\int \frac{1 + \cos^2 x}{1 + \cos(2x)}\, dx$

State why the following integral are improper and determine whether they are convergent or divergent.

1. $\displaystyle\int_0^1 \frac{1}{\sqrt{|2x-1|}}\, dx$

2. $\displaystyle\int_0^7 (7-x)^{-3/2}\, dx$

3. $\displaystyle\int_1^\infty \frac{x+3}{x^2+5x+6}\, dx$

4. $\displaystyle\int_0^1 \frac{\sin^2 x}{x^{3/2}\sqrt{1-x^2}}\, dx$

5. $\displaystyle\int_{-\infty}^\infty \frac{e^{-x^2}}{\sqrt{|x|}}\, dx$

6. $\displaystyle\int_0^\infty \frac{1-\cos^2 x}{\sqrt{x}}\, dx$

7. $\displaystyle\int_{-\infty}^4 \frac{3x}{x^2\sqrt{4-x}}\, dx$

Equation Sheet:

Pythagorean Identities:
$\sin^2\theta + \cos^2\theta = 1$
$1 + \tan^2\theta = \sec^2\theta$
$1 + \cot^2\theta = \csc^2\theta$

Double Angle Formulas:
$\cos(2\theta) = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$
$\sin(2\theta) = 2\sin\theta\cos\theta$

Trigonometric Substitution:
$\sqrt{a^2 - x^2} \implies x = a\sin x$
$\sqrt{a^2 + x^2} \implies x = a\tan x$
$\sqrt{x^2 - a^2} \implies x = a\sec x$

Derivatives:
$(\sin x)' = \cos x$
$(\cos x)' = -\sin x$
$(\tan x)' = \sec^2 x$
$(\sec x)' = \tan x \sec x$
$(\cot x)' = -\csc^2 x$
$(\csc x)' = -\cot x \csc x$
$\left(\tan^{-1} x\right)' = \dfrac{1}{1+x^2}$
$\left(\sin^{-1} x\right)' = \dfrac{1}{\sqrt{1-x^2}}$
$\left(\cos^{-1} x\right)' = \dfrac{-1}{\sqrt{1-x^2}}$

Quadratic Equation:
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Factoring Cubics:
$(x^3 \pm a^3) = (x \pm a)(x^2 \mp ax + a^2)$

Comparison Test: $\forall f, g$ cont. s.t. $\forall x \geq a$, $f(x) \geq g(x) \geq 0$: (i) if $\displaystyle\int_a^\infty f(x)\,dx < \infty \implies \int_a^\infty g(x)\,dx < \infty$, (ii) if $\displaystyle\int_a^\infty g(x)\,dx = \infty \implies \int_a^\infty f(x)\,dx = \infty$.