

Solve the following integrals

$$(1) \int e^x \sin x \, dx$$

$$(2) \int \frac{x^4 + x^3 - x^2 + 1}{(x^2 + 1)(x + 2)} \, dx$$

$$(3) \int \cos x \sqrt{1 + \sin^2 x} \, dx$$

$$(4) \int \frac{1}{1 + \sin x} \, dx$$

$$(5) \int \sin^4 x \cos^3 x \, dx$$

Determine whether the following **sequences** converge or diverge. If the sequence converges determine its limit.

$$(1) a_n = \frac{\log n}{n}$$

$$(2) a_n = \frac{n^4}{3^n}$$

$$(3) a_n = \sin(n + 1)$$

$$(4) a_n = \frac{3^n - n^2}{2^n + n^5 - n}$$

$$(5) a_n = \arctan\left(\frac{n^2 + 1}{n}\right)$$

Determine whether the following **series** converge absolutely, converge conditionally or diverge.

$$(1) \sum_{n=1}^{\infty} (-1)^n \frac{\log n}{n}$$

$$(2) \sum_{n=1}^{\infty} \frac{3^n (n+3)!}{(2n)!}$$

$$(3) \sum_{n=1}^{\infty} \sin\left(\frac{1}{n^2}\right)$$

$$(4) \sum_{n=1}^{\infty} \frac{1}{\sqrt{1+n^8}}$$

$$(5) \sum_{n=1}^{\infty} (-1)^n \frac{3^n + n}{3^{n+1} - 1}$$

Determine the Taylor series for the given function about the indicated center.

$$(1) f(x) = \frac{x}{e^{x^2}}, \quad x = 0$$

$$(2) f(x) = \log(\sqrt{1-3x}), \quad x = 0$$

$$(3) f(x) = \frac{x^2}{3-x}, \quad x = 2$$

$$(4) f(x) = (1+x)e^x, \quad x = 0$$

$$(5) f(x) = \frac{1}{x^2+1}, \quad x = 1$$