MATH 1B
Summer 2019
Exam 1
July 3, 2019

Name (Print): SOLUTIONS

SID: ____________________

This exam contains 6 pages (including this cover page) and 5 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books or any calculator on this exam. One note card of notes is permitted.

You are required to show your work on each problem on this exam. The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, please ask for scrap paper. Be sure to put name on scrap paper and indicate which problem you are working on.

Do not write in the table to the right.

Problem	Points	Score
1	10	
2	8	
3	12	
4	10	
5	10	
Total:	50	

1. (10 points) Integrate the following,

$$\int x \log^2 x \, dx$$

We need to use Integration by Parts twice:

$u = \log^2 x \qquad du = 2\log x \cdot \frac{1}{x} dx$

$dv = x\,dx \qquad v = \frac{1}{2}x^2$

$$\int x \log^2 x \, dx = \frac{1}{2}x^2 \log^2 x - \int x \log x \, dx$$

$u = \log x \qquad du = \frac{1}{x} dx$

$dv = x\,dx \qquad v = \frac{1}{2}x^2$

$$= \frac{1}{2}x^2 \log^2 x - \left[\frac{1}{2}x^2 \log x - \int \frac{1}{2}x \, dx\right]$$

$$= \frac{1}{2}x^2 \log^2 x - \frac{1}{2}x^2 \log x - \frac{1}{4}x^2$$

2. Write out how you would decompose the following proper rational functions. Do not solve for the unknown coefficients,

(a) (2 points) $\dfrac{6x+2}{3x^2-10x-8}$

$$= \frac{A}{3x+2} + \frac{B}{x-4}$$

(b) (2 points) $\dfrac{1}{(x^2-4)(x+2)}$

$$= \frac{A}{x-2} + \frac{B}{x+2} + \frac{C}{(x+2)^2}$$

(c) (2 points) $\dfrac{4x^2-7}{x^5+5x^3}$

$$= \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{Dx+E}{x^2+5}$$

(d) (2 points) $\dfrac{x-1}{(x^3+2x^2+x+2)(x-4)^3}$

$$= \frac{Ax+B}{x^2+1} + \frac{C}{x+2} + \frac{D}{x-4} + \frac{E}{(x-4)^2} + \frac{F}{(x-4)^3}$$

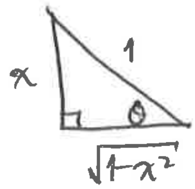
3. (12 points) Integrate the following,

$$\int x^5 \sqrt{1-x^2}\, dx$$

Use a <u>trigonometric substitution</u>:

$x = \sin\theta$

$dx = \cos\theta \, d\theta$



$$\int x^5\sqrt{1-x^2}\,dx = \int \sin^5\theta\sqrt{1-\sin^2\theta}\cdot\cos\theta\,d\theta = \int \sin^5\theta\cos^2\theta\,d\theta$$

Leave out one $\sin\theta$, convert the rest to $\cos\theta$ using the identity $\sin^2\theta = 1-\cos^2\theta$.

$$\int \sin^5\theta\cos^2\theta\,d\theta = \int (1-\cos^2\theta)^2\cos^2\theta\cdot\sin\theta\,d\theta$$

Make a <u>substitution</u>: $u=\cos\theta$ $du = \sin\theta\,d\theta\cdot(-1)$

$$= -\int (1-u^2)^2u^2\,du = -\int (u^2-2u^4+u^6)\,du$$

$$= -\frac{1}{3}u^3 + \frac{2}{5}u^5 - \frac{1}{7}u^7$$

$$= -\frac{1}{3}(1-x^2)^{3/2} + \frac{2}{5}(1-x^2)^{5/2} - \frac{1}{7}(1-x^2)^{7/2}$$

4. (10 points) Integrate the following,

$$\int \frac{\sin x \cos x}{\cos^2 x + 2\sin^2 x - 2\sin x}\, dx$$

Replace $\cos^2 x = 1 - \sin^2 x$,

$$= \int \frac{\sin x \cos x}{(\sin x - 1)^2}\, dx$$

Make a substitution: $u = \sin x$ $du = \cos x\, dx$

$$= \int \frac{u}{(u-1)^2}\, du = \int \frac{u-1}{(u-1)^2}\, du + \int \frac{1}{(u-1)^2}\, du$$

$$= \int \frac{1}{u-1}\, du + \int \frac{1}{(u-1)^2}\, du$$

you could use partial frac. decomp. to get here.

$$= \log|u-1| - \frac{1}{u-1}$$

$$= \log|\sin x - 1| - \frac{1}{\sin x - 1}$$

5. (10 points) Integrate the following,

$$\int \sqrt{\cos(2x)}\cos x \, dx$$

Replace $\cos(2x) = 1 - 2\sin^2 x$:

$$= \int \sqrt{1-2\sin^2 x}\, \cos x \, dx$$

Use a substitution: $u = \sqrt{2}\sin x$ $\quad du = \sqrt{2}\cos x \, dx$

$$= \int \sqrt{1-u^2} \cdot \frac{1}{\sqrt{2}}\, du = \frac{1}{\sqrt{2}} \int \sqrt{1-u^2}\, du$$

Now make a trig. substitution: $u = \sin\theta$ $\quad du = \cos\theta \, d\theta$

$$= \frac{1}{\sqrt{2}} \int \sqrt{1-\sin^2\theta} \cdot \cos\theta \, d\theta = \frac{1}{\sqrt{2}} \int \cos^2\theta \, d\theta$$

double ~~half~~ angle identity

$$= \frac{1}{\sqrt{2}} \int \frac{1}{2}(1+\cos 2\theta)\, d\theta$$

$$= \frac{1}{2\sqrt{2}}\theta + \frac{1}{4\sqrt{2}}\sin 2\theta$$

double angle identity

$$= \frac{1}{2\sqrt{2}}\theta + \frac{1}{2\sqrt{2}}\sin\theta\cos\theta$$

$$= \frac{1}{2\sqrt{2}}\arcsin(\sqrt{2}\sin x) + \frac{1}{2\sqrt{2}}\sqrt{2}\sin x\sqrt{1-2\sin^2 x}$$