Lec 29: Separable Equations (9.3)

For the remainder of the course we will be concerned will differential equations. Broadly, a **differential equation** is a relation between a function, it's derivatives, and the independent variables associated with it. To understand this let's consider some common examples,

1. Newton's Second Law: $F = ma$, which can be alternatively written as $F(x, t) = m\frac{dx}{dt}$.
2. Population Growth: there are many models including the exponential, $\frac{dP}{dt} = P$, and the logistic, $\frac{dP}{dt} = kP\left(1 - \frac{P}{M}\right)$.
3. Predator-Prey Models: most famous is the Lotka-Volterra model. (Equations like this are discussed in section 9.6, however we will not cover this section. Solving these explicitly are beyond the scope of this course.)

Chapter 9 deals with **first order** differential equation. A first order differential equation only involves the first derivative of the function (and no higher orders), that is

$$\frac{dy}{dx} = f(y, x) \tag{1}$$

We will start by dealing with first order **separable** differential equations. This means the differential equation can be written as,

$$\frac{dy}{dx} = f(x)g(y) \tag{2}$$

The general procedure for solving separable differential equations is,

$$\frac{dy}{dx} = f(x)g(y) \tag{3}$$

$$\frac{dy}{g(y)} = f(x)\ dx \tag{4}$$

$$\int \frac{dy}{g(y)} = \int f(x)\ dx \tag{5}$$

Equation (5) defines y implicitly as a function of x. In many cases we are able to solve for y (and should be done if able).

EXAMPLE 1. Solve for the family of functions that satisfy $\frac{dP}{dt} = P$ (exponential growth).

For most differential equations there are infinitely many functions that satisfy them. We categorize all of these functions as the **family of solutions** or **family of functions**.

EXAMPLE 2. Solve the initial value problem $\frac{dy}{dx} = \frac{x^2}{y^2}$, $y(0) = 2$.

Definition. An **initial value problem** like the one above, specifies the value of the function at a given point. For first order ODEs (ordinary differential equations) there is only one solution to an initial value problem.

EXAMPLE 3. Solve the equation $y' = x^2 y$.