Lec 26: Taylor and McLaurin Series, cont'd (11.10)

The following series will be relevant to the lecture,

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \cdots \tag{1}$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots \tag{2}$$

$$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots \tag{3}$$

$$\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots \tag{4}$$

Sometimes we can use the Taylor series definition to find the series for a given function.

EXAMPLE 1. Find the McLaurin series of $f(x) = 5^x$ using the definition.

EXAMPLE 2. Find the McLaurin series of $f(x) = 5^x$ using the McLaurin series for e^x.

Most of the time, however, it is best to manipulate Taylor series we already know, as computing derivatives can become complicated and sometimes patterns are hard to see.

EXAMPLE 3. Find the McLaurin series for $f(x) = \int e^{x^2}\, dx$.

EXAMPLE 4. Find the McLaurin series for $f(x) = \sin x \cos x$.

Sometimes we may not be able to find the Taylor series for a given functions, but we can find the first few terms using multiplication and long division of series.

EXAMPLE 5. Find the first four terms of the McLaurin series for $f(x) = e^x \sin x$.

EXAMPLE 6. Find the first four terms of the McLaurin series for $f(x) = \dfrac{e^x}{x^2+1}$.

We can also revisit function limit problems and use Taylor series to simplify the work necessary to solve the limit...

EXAMPLE 7. Evaluate $\lim\limits_{x \to 0} \dfrac{e^x - 1 - x}{x^2}$.