EXAMPLE 1. Compute the n-th derivative of the function $f(x) = \sum_{n=0}^{\infty} c_n(x-a)^n$ at $x = a$.

See that the above calculation gives us a formula for the coefficients c_n, and we get

$$c_n = \frac{f^{(n)}(a)}{n!} \tag{1}$$

where $f^{(n)}(a)$ denotes the n-th derivative of the function at $x = a$. This leads us to introduce the following theorem,

Theorem. If f has a power series representation (expansion) about a, that is, if

$$f(x) = \sum_{n=0}^{\infty} c_n(x-a)^n \; |x-a| < R \tag{2}$$

the the coefficients are given by the formula

$$c_n = \frac{f^{(n)}(a)}{n!} \tag{3}$$

This is called the **Taylor series** of f about $x = a$. For the case when $a = 0$ we use the term **McLaurin series**.

EXAMPLE 2. Derive the McLaurin series of $f(x) = e^x$ and determine the radius of convergence.

EXAMPLE 3. Derive the McLaurin series of $f(x) = \sin x$.

EXAMPLE 4. Derive the McLaurin series of $f(x) = \cos x$.

EXAMPLE 5. Show that $e^{ix} = \cos x + i \sin x$ using McLaurin series.

We will also introduce the binomial series without proof,

$$(1+x)^k = \sum_{n=0}^{\infty} \binom{k}{n} x^n = 1 + kx + \frac{k(k-1)}{2!}x^2 + \frac{k(k-1)(k-2)}{3!}x^3 + \cdots \tag{4}$$

where the large parentheses represent the binomial coefficient. We will not do much with the binomial series, however it is important to know what it is and what the binomial coefficient is.

EXAMPLE 6. Expand the polynomial $(x+1)^7$.

EXAMPLE 7. Write the McLaurin series for $(2+x)^{3/2}$.

Note that all the theorems and rules we used on the series in section 11.9 can also be applied to these series. We will practice manipulating them more next lecture.