Lec 17: The Direct Comparison Test (11.4)

In this lecture we will discuss the direct comparison test, one of two comparison tests we will learn. Next lecture we will cover the limit comparison test. The direct comparison test is similar to the comparison test for improper integrals we learned back in section 7.8.

The Comparison Test. Suppose $\sum a_n$ ad $\sum b_n$ are series with positive terms, then (i) if $\sum b_n$ is convergent and $a_n \leq b_n$ for all n, then $\sum a_n$ is also convergent, (ii) If $\sum b_n$ is divergent and $a_n \geq b_n$ for all n, then $\sum a_n$ is also divergent.
Just like the comparison test for integrals, we cannot make any conclusions for the other two cases. The rest of todays lecture will consist of examples.

EXAMPLE 1. Determine whether the series $\displaystyle\sum_{n=1}^{\infty} \frac{3}{n^2+3n}$ converges or diverges.

EXAMPLE 2. Determine whether the series $\displaystyle\sum_{n=1}^{\infty} \frac{\sin^2 n + 1}{n}$ converges or diverges.

EXAMPLE 3. Can we use the comparison test on $\sum_{n=1}^{\infty} \frac{\sin n}{n^5}$ and if so does the series converge or diverge?

EXAMPLE 4. Determine whether the series $\sum_{n=1}^{\infty} \frac{1}{n^2 \log_{25} n}$ converges or diverges.

EXAMPLE 5. Determine whether the series $\sum_{n=1}^{\infty} \frac{1}{n^2 - 3n}$ converges or diverges.

EXAMPLE 6. Determine whether the series $\displaystyle\sum_{n=1}^{\infty} \frac{4^{n+1}+n}{3^n-1}$ converges or diverges.