Definition. A **sequence** is an ordered list of numbers, denoted $\{a_1,\ a_2,\ a_3, \dots\}$, $\{a_n\}$, or $\{a_n\}_{n=1}^{\infty}$.
Notice that we can start counting a sequence whenever we want. Often times there is an explicit formula for a sequence (i.e. $a_n = 3n - n^2$), but sometimes sequences are given implicitly (i.e. $a_1 = 1$, $a_{n+1} = a_n/n!$). When dealing with sequences we are concerned with the limit of the sequence as n goes to infinity,

$$\lim_{n\to\infty} a_n \tag{1}$$

Definition. A sequence $\{a_n\}$ has a **limit** L if for every $\epsilon > 0$ there is a corresponding integer N such that for all $n > N$, $|a_n - L| < \epsilon$. We then say

$$\lim_{n\to\infty} a_n = L \tag{2}$$

You don't need to be concerned with this definition of a limit, we will not be using the formal definition in this class. It is good however to understand intuitively what a limit entails. If a sequence has a limit we say it converges, if a sequence does not have a limit (this includes the sequence going off to infinity) then we say the sequence diverges. Now we will introduce some theorems that will help us find sequence limits. We will first deal with explicit sequences.

Theorem. If $\lim\limits_{n\to\infty} f(x) = L$ and $f(n) = a_n$ when n is an integer, then $\lim\limits_{n\to\infty} a_n = L$.

When referring to the $f(x)$ detailed in the above theorem I will use the terminology "corresponding function".

EXAMPLE 1. What is the limit of the sequence $\left\{\dfrac{1}{n^2}\right\}$.

The following are a list of rules limits obey. Let $\{a_n\}$ and $\{b_n\}$ be convergent sequences and c a constant,

- $\lim\limits_{n\to\infty} (a_n \pm b_n) = \lim\limits_{n\to\infty} a_n \pm \lim\limits_{n\to\infty} b_n$
- $\lim\limits_{n\to\infty} ca_n = c \lim\limits_{n\to\infty} a_n$

- $\lim\limits_{n\to\infty}(a_n b_n) = \lim\limits_{n\to\infty} a_n \cdot \lim\limits_{n\to\infty} b_n$
- $\lim\limits_{n\to\infty} a_n/b_n = \left(\lim\limits_{n\to\infty} a_n\right) / \left(\lim\limits_{n\to\infty} b_n\right)$, if $\lim\limits_{n\to\infty} b_n \neq 0$
- $\lim\limits_{n\to\infty} a_n^p = \left(\lim\limits_{n\to\infty} a_n\right)^p$

EXAMPLE 2. Find $\lim\limits_{n\to\infty} \dfrac{n^3+n}{3+2n^2-4n^3}$

Theorem (Squeeze theorem for sequences). If $a_n \le b_n \le c_n$ for all $n \ge n_0$ and $\lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} c_n = L$, then $\lim\limits_{n\to\infty} b_n = L$.

Theorem. If $\lim\limits_{n\to\infty} |a_n| = 0$, then $\lim\limits_{n\to\infty} a_n = 0$.

EXAMPLE 3. Calculate $\lim\limits_{n\to\infty} \dfrac{\sin n}{n^2}$

EXAMPLE 4. Calculate $\lim\limits_{n\to\infty} \dfrac{\ln n}{n}$

Theorem. If $\lim_{n\to\infty} a_n = L$ and the function f is continuous at L, then $\lim_{n\to\infty} f(a_n) = f(L)$.

EXAMPLE 5. Find $\lim_{n\to\infty} \sin(\pi/n)$