Lec 7: Integration of Rational Functions by Partial Fraction, cont'd (7.4)

Let's do a quick example to refresh what was learned in the previous lecture,

EXAMPLE 1. Evaluate $\int \frac{dx}{(x-1)(x+3)^2}\,dx$

CASE 3: $Q(x)$ **contains irreducible quadratic factors, none of which is repeated.** For any factor of $Q(x)$ that is an irreducible quadratic $ax^2 + bx + c$ we include the term

$$\frac{Ax + B}{ax^2 + bx + c} \tag{1}$$

in our partial fraction decomposition. Note that the numerator of this term is one degree less than the denominator. This is the general procedure you should follow for non repeated, irreducible factors.

How do we know a quadratic is irreducible? Check the determinant. If $b^2 - 4ac < 0$ then the quadratic is irreducible.

EXAMPLE 2. Evaluate $\int \frac{dx}{x^3 + x}\,dx$

CASE 4: $Q(x)$ contains a repeated irreducible quadratic factor. If $Q(x)$ contains some factor $(ax^2+bx+c)^r$ then we follow the procedure in case 3 for the numerator, and the procedure in case 2 for the denominator of the necessary terms to add to the partial fraction decomposition. The terms we need to add have the form,

$$\frac{A_1x + B_1}{ax^2 + bx + c} + \frac{A_2x + B_2}{(ax^2 + bx + c)^2} + \cdots + \frac{A_rx + B_r}{(ax^2 + bx + c)^r} \tag{2}$$

EXAMPLE 3. Evaluate $\displaystyle\int \frac{1 - x + 2x^2 - x^3}{x(x^2+1)^2}\,dx$

<u>Note:</u> any cubic polynomial must have one real root, because complex roots come in conjugate pairs (this isa consequence of the fundamental theorem of algebra) and any quadratic you encounter in this class will be factorable (and factorable using methods you know). Thus these are all the cases we will address.

The last topic we will address in this section is what the book refers to as **rationalizing solutions**. This small sections addresses problems where after the appropriate u-substition a problem clearly becomes a partial fractions problems.

EXAMPLE 4. Evaluate $\displaystyle\int \frac{dx}{x\sqrt{x-1}}\,dx$

EXAMPLE 5. Evaluate $\int \frac{x^3}{\sqrt[3]{x^2+1}}\,dx$