Lec 5: Trigonometric Substitution (7.3)

For this section we will utilize what is sometimes called **inverse substitution**. This follows the same general procedure as substitution, except that we set $x = f(\theta)$ and so $dx = f'(\theta)$. The following table describes the necessary substitution given the type of problem:

Expression	Substitution	Identity
$\sqrt{a^2 - x^2}$	$x = a\sin\theta$	$1 - \sin^2\theta = \cos^2\theta$
$\sqrt{a^2 + x^2}$	$x = a\tan\theta$	$1 + \tan^2\theta = \sec^2\theta$
$\sqrt{x^2 - a^2}$	$x = \sec\theta$	$\sec^2\theta - 1 = \tan^2\theta$

EXAMPLE 1. Evaluate $\displaystyle\int \frac{dx}{x^2\sqrt{4 - x^2}}$

EXAMPLE 2. Evaluate $\displaystyle\int \frac{\sqrt{x^2 - 4}}{x}\,dx$

Recall a procedure that you may have learned in algebra known as **completing the square**, where we transform an arbitrary polynomial so it is in the form $(x-\alpha)^2+\beta$. We will need to be comfortable completing the square to evaluate certain integrals using trigonometric substitution.

EXAMPLE 3. Complete the square for the polynomial $x^2+4x+11$.

EXAMPLE 4. Complete the square for the polynomial $3-2x-x^2$.

EXAMPLE 5. Evaluate $\displaystyle\int x^2\sqrt{3-2x-x^2}\,dx$