Lec 4: Trigonometric Integrals (7.2)

This section consists of solving integrals comprised of trigonometric functions. Here is a list of important relations and derivatives:

1. Elementary derivatives:
   (a) $(\sin x)' = \cos x$
   (b) $(\cos x)' = -\sin x$
   (c) $(\tan x)' = \sec^2 x$
   (d) $(\sec x)' = \tan x \sec x$
   (e) $(\cot x)' = -\csc^2 x$
   (f) $(\csc x)' = -\cot x \csc x$
2. Pythagorean identities:
   (a) $\sin^2 x + \cos^2 x = 1$
   (b) $1 + \tan^2 x = \sec^2 x$
   (c) $1 + \cot^2 x = \csc^2 x$
3. Double angle formulas:
   (a) $\sin(2x) = 2\sin x \cos x$
   (b) $\cos(2x) = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$
4. Multiplication to addition formulas:
   (a) $\sin x \cos y = \frac{1}{2}(\sin(x-y) + \sin(x+y))$
   (b) $\sin x \sin y = \frac{1}{2}(\cos(x-y) - \cos(x+y))$
   (c) $\cos x \cos y = \frac{1}{2}(\cos(x-y) + \cos(x+y))$

When solving trigonometric integrals it is best to use the identities above to simplify the problem to an integral we know, or to something we may perform a u-substitution on. Sometimes we will need to use by parts as well.

EXAMPLE 1. Evaluate $\int \sin^3 x \cos x \, dx$

EXAMPLE 2. Evaluate $\int \cos^4 x \, dx$

EXAMPLE 3. Evaluate $\int \cos^2 x \sin^3 x \, dx$

Now let us summarize what to do for integrals of the form,

$$\int \sin^n x \cos^m x \, dx \tag{1}$$

Firstly, if $m = 1$ or $n = 1$ we can do an immediate u-sub. If m and n are both even (we include 0 with even) then we must use the cos double angle formula as many times as necessary. If either m or n is odd (or both) we need to use the pythagorean identity, $\sin^2\theta + \cos^2\theta = 1$ to get us to an integral where we can make a u-sub (always either $u = \cos x$ or $u = \sin x$.

Integrals of the form

$$\int \sec^n x \tan^m x \, dx \tag{2}$$

are a bit trickier to categorize and don't have a nice double angle formula to work with. We should look for these things (in this order!): (1) is there an immediate u-sub, (2) can using the pythagorean relation lead us to a u-sub, (3) using integration by parts (these often have

non-obvious steps!), and lastly (4) try writing it out in sin's and cos's.

EXAMPLE 4. Evaluate $\int \sec^6 x \, dx$

EXAMPLE 5. Evaluate $\int \tan^5 x \sec^3 x \, dx$

EXAMPLE 6. Evaluate $\int \sec^3 x \, dx$

EXAMPLE 7. Evaluate $\int \sin(3x)\cos(2x)\,dx$