Recall the product rule for differentiable functions,

$$\left(e^{3x}\sin x\right)' = 3e^{3x}\sin x + e^{3x}\cos x \tag{1}$$

We can write the general formula as such,

$$\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x) \tag{2}$$

Let's integrate both sides and see what happens,

$$\int \frac{d}{dx}[f(x)g(x)]\,dx = \int [f(x)g'(x) + g(x)f'(x)]\,dx \tag{3}$$

$$f(x)g(x) = \int f(x)g'(x)\,dx + \int g(x)f'(x)\,dx \tag{4}$$

Integration by Parts (IBP) Formula. Rearranging the above and set $u = f(x)$ and $v = g(x)$. Thus by substitution $du = f'(x)dx$ and $dv = g'(x)dx$ and the above formula becomes,

$$\int u\,dv = uv - \int v\,du \tag{5}$$

When attempting a problem that is integration by parts we want to guess the correct choices for u and dv and from there differentiate to find du and integrate to find v. Here are some hints for the process (in no particular order):

(1) We should be able to integrate dv without too much effort.
(2) Sometimes $dv = dx$.
(3) Sometimes we need to use by parts more than once.
(5) In general, polynomials are u.
(5) If your new integral is <u>harder</u> than your original you probably want to start over.
(6) We might want to use a substitution first!

EXAMPLE 1. Evaluate $\int xe^x\,dx$

EXAMPLE 2. Evaluate $\int \ln(x)\, dx$

EXAMPLE 3. Evaluate $\int (x^2 + 3x) \sin(x)\, dx$