Lec 1: The Substitution Rule (5.5)

This is review from math 1a, but this topic in particular will carry over heavily into math 1b and is necessary for success in the course. You should expect the substitution rule to play a role in many integrals you will be expected to solve in the first third of the course.

The Substitution Rule. If $u = g(x)$ is a differentiable function whose range is an interval I and f is continuous on I, then

$$\int f\left(g(x)\right) g'(x)\, dx = \int f(u)\, du \tag{1}$$

The best way to understand the substitution rule is through practice:

EXAMPLE 1. Evaluate $\int x\sqrt{3+x^2}\, dx$

EXAMPLE 2. Evaluate $\int \tan(x)\, dx$

EXAMPLE 3. Evaluate $\displaystyle\int \frac{2x+1}{x^2+x-5}\,dx$

What if we have a **definite integral** (recall a definite integral means the endpoints are defined)? We must be extra careful when evaluating and either (1) change bounds of integration to in terms of u, (2) change the integral back to the original variable before evaluating at endpoints.

It is usually best to pick option (1), however (2) is sometimes easier when dealing with trigonometric or other difficult functions.

The Substitution Rule for Definite Integrals. If g' is continuous on $[a, b]$ and f is continuous on the range of $u = g(x)$, then

$$\int_a^b f\left(g(x)\right) g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du \tag{2}$$

EXAMPLE 4. Evaluate $\displaystyle\int_0^4 xe^{x^2}\,dx$