

Determine whether the following series converge absolutely, converge conditionally, or diverge. Be sure to clearly state what test you are using and the conclusion you come to.

$$(1) \sum_{n=1}^{\infty} \frac{3^n - 2^n}{4^n}$$

$$(2) \sum_{n=1}^{\infty} (-1)^{n+1} \frac{\log n}{n^2}$$

$$(3) \sum_{n=1}^{\infty} (-1)^n \frac{n^2 + 3}{2n^2 - 1}$$

$$(4) \sum_{n=1}^{\infty} \frac{2^n (2n)!}{(n+5)!}$$

$$(5) \sum_{n=1}^{\infty} \frac{n^n}{n!}$$

$$(6) \sum_{n=1}^{\infty} (-1)^n \sqrt[n]{2 - \frac{1}{n}}$$

Compute the radius and interval of convergence for the following power series.

$$(1) \sum_{n=1}^{\infty} (-1)^n x^n$$

$$(2) \sum_{n=1}^{\infty} n!(x-2)^n$$

$$(3) \sum_{n=1}^{\infty} \frac{1}{2^n} (x+1)^n$$

$$(4) \sum_{n=1}^{\infty} \frac{3^n}{2n+1} (x-1)^n$$

Determine the Taylor series for the given function about the indicated center, using any method of your choosing.

(1) $f(x) = e^{3x}$, $x = 0$

(2) $f(x) = \frac{1}{2-x}$, $x = 1$

(3) $f(x) = \sin(x) \cos(x)$, $x = 0$

(4) $f(x) = \log(1 + x^2)$, $x = 0$

(5) $f(x) = e^{x^3}$, $x = 1$

(6) $f(x) = \tan x$, $x = 0$

Compute the sum of the convergent series using your knowledge of Taylor series.

$$(1) \sum_{n=0}^{\infty} (-1)^{n+1} \frac{1}{n+1}$$

$$(2) \sum_{n=0}^{\infty} \frac{(-1)^n}{n!}$$

$$(3) \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!}$$

$$(4) \sum_{n=0}^{\infty} \frac{3^n}{n!}$$