Lecture 18: notes

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The direct comparison test

Just like there is a comparison test for integral there is also one for series,

<u>The comparison test</u>. Suppose that $\sum a_n$ and $\sum b_n$ are series with non-negative terms.

(i) If $\sum b_n$ is convergent and $a_n \le b_n$ for all n, then $\sum a_n$ also converges.

(ii) If $\sum b_n$ is divergent and $b_n \le a_n$ for all n, then $\sum a_n$ also diverges.

<u>Ex</u> 1. Does $\sum_{n=1}^{\infty} \frac{1}{2^n+1}$ converge or diverge?

This looks very similar to

$$\sum_{n=1}^{\infty} \frac{1}{2^n} = \sum_{n=1}^{\infty} \frac{1}{2}\left(\frac{1}{2}\right)^{n-1}$$

geometric with $r=\frac{1}{2}$ thus conv.

So let's use the comparison test,

$$\frac{1}{2^n+1} \le \frac{1}{2^n} \quad \text{for all } n$$

$$\sum_{n=1}^{\infty} \frac{1}{2^n} \text{ conv.} \Rightarrow \sum_{n=1}^{\infty} \frac{1}{2^n+1} \text{ conv.}$$ by comparison test.

Ex2. Does $\sum_{n=1}^{\infty} \frac{\log n}{n}$ converge or diverge?

Let's compare the above to the harmonic series,

$$\frac{\log n}{n} \geq \frac{1}{n} \quad \text{as long as } \log n > 1 \Rightarrow n > 3.$$

comp test works as long as this some finite #.

$$\sum_{n=1}^{\infty} \frac{1}{n} \text{ div.} \Rightarrow \sum_{n=1}^{\infty} \frac{\log n}{n} \text{ div}$$

(by comparison test)

What should we do:

1. Guess conv. or div. based on "dominant terms"
2. Determine inequality
3. State comparison test.

Ex3. Does $\sum_{n=1}^{\infty} \frac{3n}{2n^2-n}$ converge or diverge?

1. the dominant terms are $\frac{3n}{2n^2} = \frac{3}{2n}$ so my guess is that the series diverges (because $\sum \frac{1}{n}$ diverges).

2. $\frac{3n}{2n^2-n} \geq \frac{3n}{2n^2} = \frac{3}{2n} \geq \frac{1}{n}$

3. $\sum_{n=1}^{\infty} \frac{1}{n}$ div. $\Rightarrow \sum_{n=1}^{\infty} \frac{3n}{2n^2-n}$ div. by comp. test.