Practice Problems

1. Prove or disprove the following:
   
   (a) \( \exists x (P(x) \lor Q(x)) \equiv \exists x P(x) \lor \exists x Q(x) \)
   
   (b) \( \exists x (P(x) \land Q(x)) \equiv \exists x P(x) \land \exists x Q(x) \)

2. Let \( f : A \rightarrow B \) be a function and let \( S \subseteq B \). Show that \( f^{-1}(\overline{S}) = \overline{f^{-1}(S)} \).

3. Let \( d \) be a fixed positive integer. Let \( f : \mathbb{Z} \rightarrow \mathbb{Z} \) be the function that takes \( a \) to \( a \text{ div } d \) and \( g : \mathbb{Z} \rightarrow \mathbb{Z} \) be the function that takes \( a \) to \( a \text{ mod } d \). Are either of these functions one-to-one? Onto?

4. Show that \( \mathbb{Z}^+ \times \mathbb{Z}^+ \) is countable.

5. Prove that there are infinitely many primes.