Concepts

set	power set	complement	range
element	Cartesian product	function	image/preimage
$\mathbb{R}, \mathbb{Q}, \mathbb{Z}, \mathbb{N}, \emptyset$	union	domain	injective
cardinality	intersection	codomain	surjective

Practice Problems

1. Find the power sets of the following sets.

(a) $\{\{1, 2\}\}$

(b) $\{1, \{2, 3\}\}$

(c) $\{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}$

2. Let A be the set of Cal student who like rain, B be the set of Cal students who have never seen snow, and C the set of Cal students who wish they lived by the beach. Describe the students in each of the following combinations of sets and draw the corresponding Venn diagram.

(a) $A \cap (B - C)$

(b) $\overline{A} \cup \overline{B} \cup \overline{C}$

(c) $B - (A \cap \overline{C})$

3. Are $A \times B \times C$ and $(A \times B) \times C$ equal? Why or why not? What about $A \cup (B \cup C)$ and $(A \cup B) \cup C$?

4. (De Morgan's laws) Show that $\overline{A \cap B} = \overline{A} \cup \overline{B}$ by showing that each set contains the other.

5. Determine if these functions are injective or surjective.

(a) $f : \mathbb{R} \to \mathbb{Q}$ where $f(x) = \lceil x \rceil$

(b) $g : \mathbb{Z} \to \mathbb{Q}$ where $g(x) = \lceil x \rceil$

(c) $h : \mathbb{R} \to \mathbb{Z}$ where $h(x) = \lceil x \rceil$

Are f and h the same function?

6. Let f be the function that assigns to each bit string the number of 1's in the string minus the number of 0's. Find the domain (i.e. the set of elements assigned a value by f) of f and a possible codomain. What choice of codomain makes f onto? Is f one-to-one?

7. (Bonus problem) Let A and B be finite sets of the same cardinality. Show that a function $f : A \to B$ is injective if and only if it is surjective.