Concepts

<table>
<thead>
<tr>
<th>set</th>
<th>power set</th>
<th>complement</th>
<th>range</th>
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<tbody>
<tr>
<td>element</td>
<td>Cartesian product</td>
<td>union</td>
<td>domain</td>
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<tr>
<td>$\mathbb{R}, \mathbb{Q}, \mathbb{Z}, \mathbb{N}, \emptyset$</td>
<td></td>
<td>intersection</td>
<td>codomain</td>
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Practice Problems

1. Find the power sets of the following sets.
   (a) $\{\{1, 2\}\}$
   (b) $\{1, \{2, 3\}\}$
   (c) $\{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}$

2. Let $A$ be the set of Cal student who like rain, $B$ be the set of Cal students who have never seen snow, and $C$ the set of Cal students who wish they lived by the beach. Describe the students in each of the following combinations of sets and draw the corresponding Venn diagram.
   (a) $A \cap (B - C)$
   (b) $\overline{A} \cup B \cup C$
   (c) $B - (A \cap \overline{C})$

3. Are $A \times B \times C$ and $(A \times B) \times C$ equal? Why or why not? What about $A \cup (B \cup C)$ and $(A \cup B) \cup C$?

4. (De Morgan’s laws) Show that $\overline{A \cap B} = \overline{A} \cup \overline{B}$ by showing that each set contains the other.

5. Determine if these functions are injective or surjective.
   (a) $f : \mathbb{R} \to \mathbb{Q}$ where $f(x) = \lceil x \rceil$
   (b) $g : \mathbb{Z} \to \mathbb{Q}$ where $g(x) = \lceil x \rceil$
   (c) $h : \mathbb{R} \to \mathbb{Z}$ where $h(x) = \lceil x \rceil$

   Are $f$ and $h$ the same function?

6. Let $f$ be the function that assigns to each bit string the number of 1’s in the string minus the number of 0’s. Find the domain (i.e. the set of elements assigned a value by $f$) of $f$ and a possible codomain. What choice of codomain makes $f$ onto? Is $f$ one-to-one?

7. (Bonus problem) Let $A$ and $B$ be finite sets of the same cardinality. Show that a function $f : A \to B$ is injective if and only if it is surjective.