Concepts

argument	direct proof
premise	proof by contraposition
conclusion	proof by contradiction
rules of inference	wlog

Practice Problems

1. Consider the argument "If n is a real number such that $n > 1$, then $n^2 > 1$. Suppose that $n^2 > 1$. Then $n > 1$." Is it valid? What if the last two sentences were instead "Suppose that $n^2 \leq 1$. Then $n \leq 1$."?

2. Use the rules of inference to go from the premises $(p \wedge t) \to (r \vee s)$, $q \to (u \wedge t)$, $u \to p$, and $\neg s$ to the conclusion $q \to r$.

3. Decide if the following arguments are valid or not. If they are, identify the rules of inference used. If they aren't, determine the logical error.

(a) Puppies are small. The dog outside of Evans is not a puppy. So the dog outside of Evans is not small.

(b) All Berkeley students can reach San Francisco by BART. Some Berkeley student has never been to San Francisco. So someone who can reach San Francisco by public transit has never been there.

(c) There is one ice cream flavor at Ici that I don't like. I taste all of the flavors every time I go to Ici. Tasting ice cream I don't like makes me sad. Therefore, I am sad every time I go to Ici.

4. Use a direct proof to show that every odd integer is the difference of 2 squares.

5. Prove or disprove that the product of a nonzero rational number and an irrational number is irrational.

6. (Triangle inequality) Prove that for x and y real numbers, $|x| + |y| \geq |x + y|$.

7. (Pigeonhole principle) Prove that if you have $n + 1$ pigeons and n holes, then at least one hole has two pigeons in it. What is the smallest number m of pigeons you can have to guarantee that one hole has 5 pigeons in it?