Concepts

predicate	universal quantifier	argument
propositional function	existential quantifier	premise
quantifier	uniqueness quantifier	conclusion
domain	De Morgan's laws	rules of inference

Practice Problems

1. Let $P(x)$ be the statement "x lives north of Dwight," where the domain of x consists of all students in your small group. Find the truth values of $P(x)$ for all x in the domain.

2. Choose 3 propositional functions $F(j)$, $G(j)$, and $H(j)$ where the domain is all dogs. Translate the following statements into English and find their truth value. (Hint: to translate into English, you may want to rewrite the statement using logical equivalences.)

(a) $\forall j F(j)$

(b) $\exists j(G(j) \rightarrow H(j))$

(c) $\forall j((F(j) \vee H(j)) \rightarrow \neg G(j))$

(d) $\exists j(\neg F(j) \wedge G(j)) \vee \forall j H(j))$

3. Let $S(x, y)$ be the statement ”x has asked y a question” and let $P(x)$ be the statement ”x is a student,” where the domain is all Math 55 GSIs and students and Professor Williams. Translate the following statements into symbols.

(a) Professor Williams has asked Melissa a question.

(b) Professor Williams has asked everyone a question.

(c) At least one student has asked another student a question.

(d) Nobody has asked every student a question.

(e) There is a student who has never been asked a question.

4. Show that $\forall x P(x) \vee \forall x Q(x)$ and $\forall x(P(x) \vee Q(x))$ are not logically equivalent.

5. Express the negations of each of these statements so that all negation symbols are in front of predicates. (Hint: remember De Morgan's Laws)

(a) $\neg(\forall x \exists y P(x, y) \ \wedge \ \forall x \exists y Q(x, y))$

(b) $\neg \exists y(S(y) \ \vee \ \forall x \neg R(x, y))$

6. Consider the argument "If n is a real number such that $n > 1$, then $n^2 > 1$. Suppose that $n^2 > 1$. Then $n > 1$." Is it valid? What if the last two sentences were instead "Suppose that $n^2 < 1$. Then $n < 1$."?