Math 1B
Quiz 6

Clearly state the tests you use and the conclusions you come to. Answers lacking justification will not receive full credit.

1. Suppose $\sum_{n=0}^{\infty} a_n x^n$ converges when $x = 3$ and diverges when $x = -5$.
   a) What can we conclude about $\sum_{n=0}^{\infty} a_n 5^n$? Why?

   b) What can we conclude about $\sum_{n=0}^{\infty} (-1)^n a_n 2^n$? Why?

2. Let $g(x) = \ln(1 + x^2)$. Given that $g(x) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^{2n}}{n}$ for $|x| < 1$, find a power series representation of $\frac{x}{1 + x^2}$. (Hint: what is $g'(x)$?)
3. Find the interval of convergence of
\[
\sum_{n=1}^{\infty} \frac{4^n (x - 2)^n}{\sqrt[3]{n} \cdot 3^n}.
\]