Find the solution of the DEs.

1. \( xy' - 2y = x^2 \)  
   \[ x 
eq 0 \]

   **Soln:** While this is a first order linear DE, it is not in the correct form, so we divide through by \( x \):
   
   \[
   y' - \frac{2}{x}y = x 
   \]
   
   \[
   y' - \frac{2}{x}y = x 
   \]
   
   \[
   P(x) = -\frac{2}{x} 
   \]
   
   \[
   \int P(x) \, dx = -2 \ln(x) 
   \]
   
   \[
   e^{\int P(x) \, dx} = e^{-2 \ln(x)} = \frac{1}{x^2} \] using ln rules

   So the integrating factor is \( \frac{1}{x^2} \).

   Multiplying through by the integrating factor gives
   
   \[
   y' x^{-2} - 2x^{-3}y = x^{-1} 
   \]
   
   \[
   \frac{d}{dx}(y x^{-2}) = x^{-1} 
   \]

   Integrating both sides:
   
   \[
   \int \frac{d}{dx}(y x^{-2}) \, dx = \int x^{-1} \, dx 
   \]
   
   \[
   y x^{-2} = \ln(x) + C 
   \]
   
   \[
   y = x^2 \ln(x) + Cx^2 
   \]

2. \( xy' + y = x \ln x \)

   **Soln:** Again, we divide through by \( x \) to get the DE in the correct form
   
   \[
   xy' + y = x \ln x 
   \]
   
   \[
   P(x) = \frac{1}{x} \], so \( \int P(x) \, dx = \ln x 
   \]
   
   \[
   e^{\int P(x) \, dx} = e^{\ln x} = x \], so the integrating factor is \( x \).

   Multiplying through by the integrating factor gives
   
   \[
   xy' + y = x \ln x 
   \]
   
   \[
   \frac{d}{dx}(xy) = x \ln x 
   \]

   Integrating both sides gives
   
   \[
   xy = \int x \ln x \, dx 
   \]
To evaluate $\int x \ln x \, dx$, we integrate by parts.

$u = \ln x, \quad dv = x \, dx$
$du = \frac{1}{x} \, dx, \quad v = \frac{x^2}{2}$

\[
\int x \ln x \, dx = \frac{x^2}{2} \ln x - \frac{1}{2} \int x \, dx
\]

\[
= \frac{x^2}{2} \ln x - \frac{x^2}{4} + C
\]

So

\[
x = \frac{x^2}{2} \ln x - \frac{x^2}{4} + C
\]

\[
y = \frac{x^2}{2} \ln x - \frac{x}{4} + C
\]