MATH 1A
QUIZ 8

Instructions: Please simplify your answers as much as possible, and box or circle your final answer. Remember, answers without justification will not receive full credit. You have 20 minutes. Good luck!

1. a) Find the domain of \( f(t) = \frac{t^2}{7t^2 - 28t - 84} \).
   
   \[ f(t) = \frac{t^2}{7(t^2 - 4t - 12)} = \frac{t^2}{7(t-6)(t+2)} \]
   
   Domain: \( t \neq 6, -2 \)

   b) Find the critical numbers of \( f(t) \).
   
   Critical numbers are \( t \) in domain of \( f \) where \( f'(t) = 0 \) or \( f' \) does not exist.

   \[ f'(t) = \frac{1}{7} \left( \frac{2t(t^2 - 4t - 12) - t^2(2t - 4)}{(t-6)^2(t+2)^2} \right) = \frac{1}{7} \left( \frac{t(2t^2 - 8t - 24 - 2t^2 + 4t)}{(t-6)^2(t+2)^2} \right) \]
   
   \[ = \frac{1}{7} \frac{t(4t - 24)}{(t-6)^2(t+2)^2} = \frac{4t(t+6)}{7(t-6)^2(t+2)^2} \]

   \( f'(t) = 0 \) if \( t = 0, -6 \). \( f'(t) \) does not exist if \( t = 6, -2 \). So critical numbers are \( t = 0, -6 \).

2. The growth rate of a bacteria population is \( \frac{dP}{dt} = 437P \). If at \( t = 0 \), there are 12 bacteria, find an expression for \( P(t) \).

   \[ P(t) = 12 e^{437t} \]

3. Does there exist a continuous differentiable function \( f \) such that \( f(1) = 4 \), \( f(3) = 5 \), and \( f'(x) > \frac{1}{2} \) for all \( x \)?
   
   No; because by the mean value theorem, there is a \( c \) in \( (1, 3) \) such that \( f'(c) = \frac{f(3) - f(1)}{3 - 1} = \frac{1}{2} \). So there is definitely an \( x \)-value such that \( f'(x) \) is not greater than \( \frac{1}{2} \).