**MATH 1A**

**QUIZ 7**

**Instructions:** Please simplify your answers as much as possible, and box or circle your final answer. Remember, answers without justification will not receive full credit. You have 20 minutes. Good luck!

1. \( \ln(xy) = \ln \left( \frac{x+y}{xy} \right) \). Find \( \frac{dy}{dx} \).

\[
\ln(xy) = \ln \left( \frac{x+y}{x+y} \right) = \ln (x+y) - \ln (x+y) \\
2 \ln(xy) = \ln(x+y) \\
2 \left( \ln(x+y) \right) = \ln(x+y)
\]

2. \( 2 \ln(x+y) = \ln(x+y) \).

\[
\frac{2}{x+y} \cdot \frac{dy}{dx} = \frac{1}{x+y} \left( 1 + \frac{dy}{dx} \right).
\]

3. Find the derivative of \( f(x) = x^\frac{1}{2} \).

\[
y = x \Rightarrow \ln(y) = \ln(x) \\
\frac{d}{dx} \left( \ln(y) \right) = x \frac{dy}{dx} \ln(x) = \frac{1}{x} \ln(x) \\
\frac{dy}{dx} = x \frac{1}{x} \left( \frac{1}{x^2} + \frac{1}{x^2} \right) = x^\frac{1}{2} \left( \frac{1 - \ln x}{x^2} \right)
\]

3. A spring with a box on the end of it hangs from the ceiling of a classroom in LeConte Hall. A bored student pulls on the box and lets go. The distance of the box from the floor is \( d(t) = e^{-t+\pi} \sin(t-\pi) + 6 \). When does the box change direction?

The box **change** direction when its velocity changes sign (i.e. where velocity = 0)

Velocity:

\[
d'(t) = -e^{-t+\pi} \sin(t-\pi) + e^{-t+\pi} \cos(t-\pi)
\]

\[
d''(t) = 0 \Rightarrow e^{-t+\pi} \left( \cos(t-\pi) - \sin(t-\pi) \right) = 0
\]

\[
\cos(t-\pi) = \sin(t-\pi)
\]

Which happens when \( t-\pi = \frac{\pi}{4} + n\pi \) for some integer \( n \).