MATH 1A
QUIZ 7

Instructions: Please simplify your answers as much as possible, and box or circle your final answer. Remember, answers without justification will not receive full credit. You have 20 minutes. Good luck!

1. $\ln(xy) = \ln\left(\frac{x+y}{xy}\right)$. Find $\frac{dy}{dx}$.

$\ln(xy) = \ln\left(\frac{x+y}{xy}\right) = \ln(x+y) - \ln(xy)$

so $2\ln(xy) = \ln(x+y)$

$\parallel$

$2(\ln(x) + \ln(y)) = \ln(x+y)$.

$\frac{d}{dx}(2(\ln(x)+\ln(y))) = \frac{d}{dx}(\ln(x+y))$

$2\left(\frac{1}{x} + \frac{1}{y}\frac{dy}{dx}\right) = \frac{1}{x+y}\left(1 + \frac{dy}{dx}\right)$

$2xy(x+y)\cdot\left(\frac{1}{x} + \frac{1}{y}\frac{dy}{dx}\right) = \left(\frac{1}{x+y} + \frac{1}{x+y}\frac{dy}{dx}\right)\cdot xy(x+$

$2y(x+y) + 2x(x+y)\frac{dy}{dx} = xy + xy\frac{dy}{dx}$

$\frac{dy}{dx}\left(2x(x+y) - xy\right) = xy - 2y(x+y)$

$\frac{dy}{dx} = \frac{xy - 2y(x+y)}{2x(x+y) - xy} = \frac{y(x - 2x - 2y)}{x(2x+2y-y)}$

$= \frac{y(-x-2y)}{x(2x+y)} = -\frac{y(2}{x(2x}$

2. Find the derivative of $f(x) = x^{\frac{1}{x}}$.

$y = x^{1/x} \Rightarrow \ln(y) = \frac{1}{x}\ln(x)$

$\frac{d}{dx}(\ln(y)) = \frac{d}{dx}\left(\frac{1}{x}\ln(x)\right) = \frac{d}{dx}\left(x^{-1}\ln(x)\right)$

$\parallel$

$\frac{1}{y}\frac{dy}{dx} = \left(-x^{-2}\ln(x) + x^{-1}\left(\frac{1}{x}\right)\right) = \left(-\frac{\ln x}{x^2} + \frac{1}{x^2}\right)$

$\frac{dy}{dx} = y\left(\frac{1-\ln x}{x^2}\right) = x^{\frac{1}{x}}\left(\frac{1-\ln x}{x^2}\right)$

3. A spring with a box on the end of it hangs from the ceiling of a classroom in LeConte Hall. A bored student pulls on the box and lets go. The distance of the box from the floor is $d(t) = e^{-t+\pi}\sin(t-\pi) + 6$. When ~~does~~ could the box change direction?

The box ~~changes~~ could change direction when its velocity changes ~~from~~ sign. (i.e. where velocity $= 0$)

velocity: $d'(t) = -e^{-t+\pi}\sin(t-\pi) + e^{-t+\pi}\cos(t-\pi)$

$= e^{-t+\pi}\left(\cos(t-\pi) - \sin(t-\pi)\right)$

$d'(t) = 0 = \cancel{\text{das}}\ \underbrace{e^{-t+\pi}}_{\text{always positive}}\left(\cos(t-\pi) - \sin(t-\pi)\right)$

so $d'(t) = 0$ if

$\cos(t-\pi) - \sin(t-\pi) = 0$

$\Updownarrow$

$\cos(t-\pi) = \sin(t-\pi)$

which happens when $t - \pi = \frac{\pi}{4} + n\pi$ (n integer)

$t = \frac{\pi}{4} + n\pi - \pi = \frac{\pi}{4} + m\pi$ (m integer)