MATH 1A
QUIZ 10

Instructions: Please simplify your answers as much as possible, and box or circle your final answer. Remember, answers without justification will not receive full credit. You have 20 minutes. Good luck!

1. You’re designing a cylindrical can (with a top) to hold the new energy drink you’ve created. You want the can to hold 120 cm$^3$ of liquid, and you want to use as little material as possible. Find the dimensions of the can that uses the smallest amount of material to construct.

\[ V = \pi r^2 h = 120 \]
\[ A = 2\pi r^2 + 2\pi rh \]
\[ h = \frac{120}{\pi r^2} \]
\[ A = 2\pi r^2 + 2\pi r \left( \frac{120}{\pi r^2} \right) \]
\[ = 2\pi r^2 + \frac{240}{r} \]

\[ \frac{dA}{dr} = 4\pi r + 240\left( -\frac{1}{r^2} \right) \]
\[ \frac{dA}{dr} = 0 \Rightarrow 4\pi r = 240 \]
\[ r^3 = \frac{60}{\pi} \]
\[ r = \left( \frac{60}{\pi} \right)^{\frac{1}{3}} \]

2. If $f'(x) = \sin x (\sec^2 x + \csc x)$ and $f(0) = 0$, find $f(x)$.

\[ f'(x) = \sin x (\frac{1}{\cos^2 x} + \frac{1}{\sin x}) = \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x} + 1 = \sec x \tan x + 1. \]

\[ f(x) = \sec x + x + C \]
\[ f(0) = 1 + C = 0 \Rightarrow C = -1 \]

\[ f(x) = \sec x + x - 1 \]

3. Find the most general antiderivative of

\[ \frac{x + 4x^2 \cos x}{x^2} \quad (x > 0). \]

\[ f(x) = \frac{x + 4x^2 \cos x}{x^2} = \frac{1}{x} + 4\cos x \]

The general antiderivative of $f$ is $\ln(x) + 4\sin x + C$. 