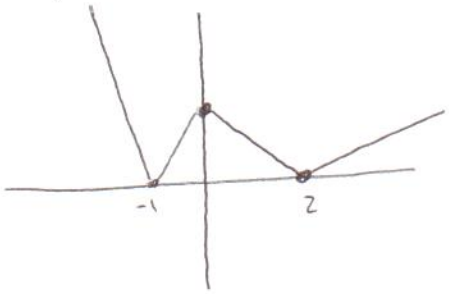
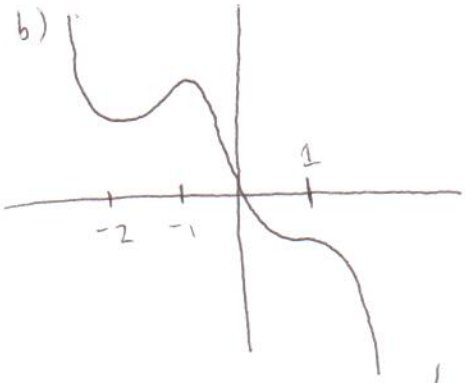
MATH 1A
Problem Solutions 10/4

1. If the graph of f is as given below, draw the graph of f?

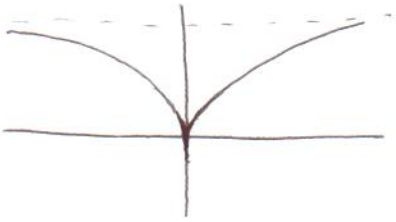
a)



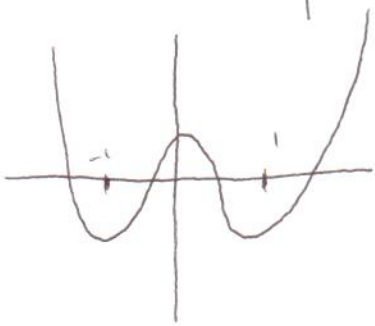
b)



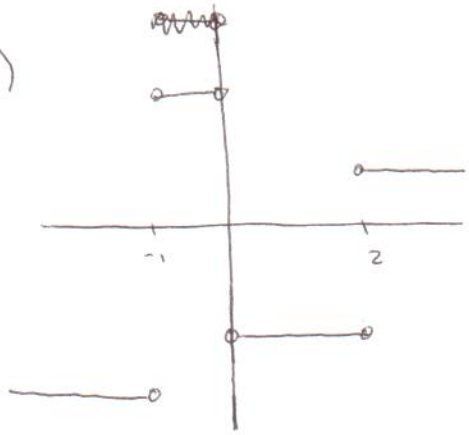
c)



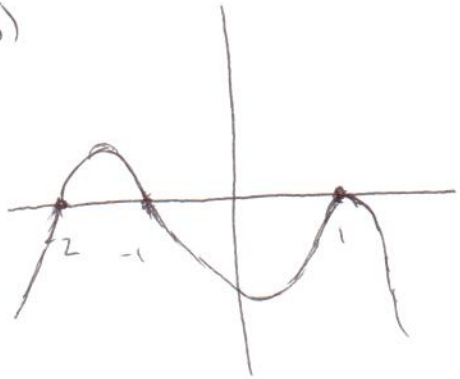
d)



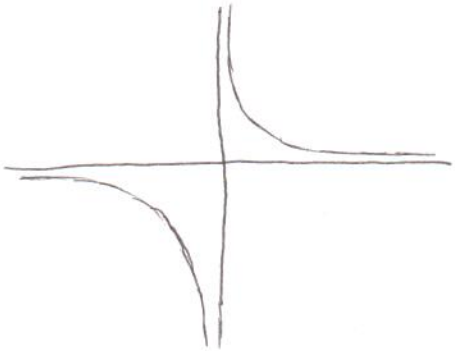
Soln: a)



b)



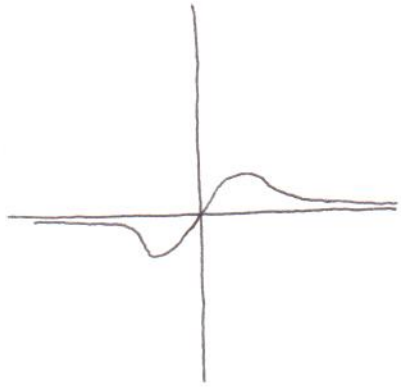
c)



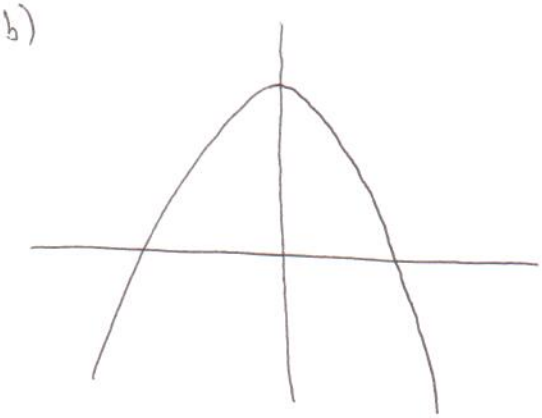
d)

2. If the graph of f' is given below, draw the graph of f.

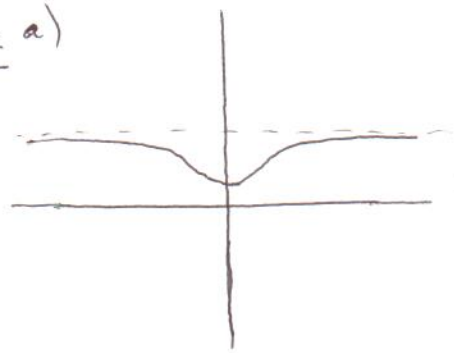
a)



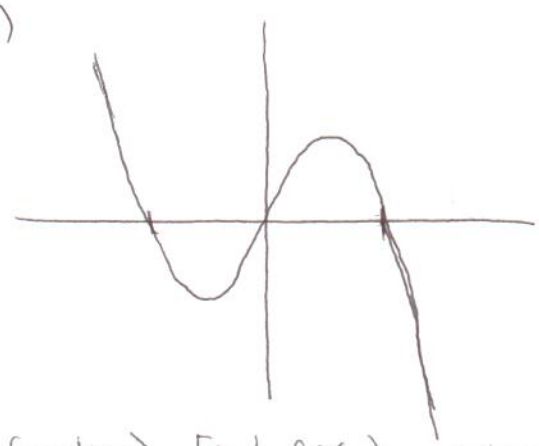
b)



Soln: a)

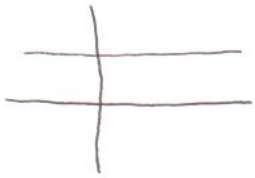


b)



3. Let $f(x)=c$ (i.e. $f(x)$ is a constant function). Find $f'(x)$ using
a) The graph of f b) the limit definition of the derivative c) the power rule & constant

Soln: a)



f looks like this. Any tangent line has slope zero, so $f'(x)=0$

b) $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{c-c}{h} = \lim_{h\to 0} 0 = 0.$

c) $f(x)=c=cx^0$ so $f'(x) = d/dx(cx^0) = c\, d/dx(x^0) = c(0\cdot x^{-1}) = 0.$

4. Pretend you only know the derivative of constant functions & the product rule. (Re)Discover the constant multiple rule.

5. Find the derivative of $\frac{2x^4+5x^3+4}{x^2}$ (Hint: choose your ~~rule~~ strategy wisely)

Soln: Easiest way: $\frac{2x^4+5x^3+4}{x^2} = 2x^2+5x+4x^{-2}$

$$d/dx(2x^2+5x+4x^{-2}) = 4x+5+(-2)4x^{-3}$$
$$= 4x+5-\frac{8}{x^3}.$$

Could also do quotient rule or product rule (with $(2x^4+5x^3+4)x^{-2}$).

6. Find the derivative of $e^x x^{1/2} + 7e^{x+1} x^3 + e^x$

Soln: $e^x x^{1/2} + 7e^{x+1} x^3 + e^x = e^x(x^{1/2} + 7e x^3 + 1)$

$$d/dx\left(e^x(x^{1/2} + 7e x^3 + 1)\right) = e^x(x^{1/2} + 7e x^3 + 1) + e^x\left(\tfrac{1}{2} x^{-1/2} + 21e x^2\right)$$ (using product rule)

7. Find the derivative of $x^{5/7}(2x+1)^2$.

Soln: $x^{5/7}(2x+1)^2 = x^{5/7}(4x^2 + 4x + 1)$

$$= 4x^{19/7} + 4x^{12/7} + x^{5/7}$$

$$d/dx\left(4x^{19/7} + 4x^{12/7} + x^{5/7}\right) = 4 \cdot 19/7\, x^{12/7} + 4 \cdot 12/7\, x^{5/7} + 5/7\, x^{-2/7}$$

8. Find the derivative of $\dfrac{e^x x^5 + x^2}{(x+1)^2}$.

Soln: $$d/dx\left(\frac{e^x x^5 + x^2}{(x+1)^2}\right) = \frac{d/dx(e^x x^5 + x^2) \cdot (x+1)^2 - (e^x x^5 + x^2) \cdot d/dx((x+1)^2)}{(x+1)^4}$$ $(x+1)^2 = x^2 + 2x + 1$

$$= \frac{[e^x x^5 + 5e^x x^4 + 2x](x+1)^2 - (e^x x^5 + x^2)(2x+2)}{(x+1)^4}$$

$$= \frac{(x+1)\left[(e^x x^5 + 5e^x x^4 + 2x)(x+1) - (e^x x^5 + x^2)(2)\right]}{(x+1)^4}$$

$$= \frac{e^x x^6 + 5e^x x^5 + 2x^2 + e^x x^5 + 5e^x x^4 + 2x - 2e^x x^5 - 2x^2}{(x+1)^3}$$

$$= \frac{e^x x^6 + 4e^x x^5 + 5e^x x^4 + 2x}{(x+1)^3} = \frac{x(e^x x^5 + 4e^x x^4 + 5e^x x^3 + 2)}{(x+1)^3}$$