MATH 1A
Solutions 10/27

2. Find the following limits:

1. $\lim_{x \to 4} \frac{x^2 - 6x + 8}{x^2 - 3x - 4}$

Soln: This is of the form $\frac{0}{0}$, so can use l'Hospital's rule. (Note: You can also factor top & bottom & simplify).

$$\lim_{x \to 4} \frac{x^2 - 6x + 8}{x^2 - 3x - 4} = \lim_{x \to 4} \frac{2x - 6}{2x - 3} = \frac{8-6}{8-3} = \frac{2}{5}.$$

2. $\lim_{x \to \infty} \frac{x^4 + 3x^2 + 4}{x^4 + 12x}$

Soln: It's easier to just evaluate this limit by multiplying top & bottom by $\frac{1}{x^4}$, but it's also of the form $\frac{\infty}{\infty}$, so we could use l'Hospital's.

$$\lim_{x \to \infty} \frac{x^4 + 3x^2 + 4}{x^4 + 12x} = \lim_{x \to \infty} \frac{4x^3 + 6x}{4x^3 + 12}$$ ← still of the form $\frac{\infty}{\infty}$, so use l'Hospital's again.

$$= \lim_{x \to \infty} \frac{12x^2 + 6}{12x^2}$$ ← still of the form $\frac{\infty}{\infty}$, use l'H's again

$$= \lim_{x \to \infty} \frac{24x}{24x} = \lim_{x \to \infty} 1 = 1.$$

3. $\lim_{x \to 0} \frac{\tan 3x}{\sin 4x}$

Soln: This is of the form $\frac{0}{0}$, so we can use l'H's:

$$\lim_{x \to 0} \frac{\tan 3x}{\sin 4x} = \lim_{x \to 0} \frac{3 \sec^2 3x}{4 \cos 4x} = \frac{3}{4}$$

4. $\lim_{x \to 0} \dfrac{1-2^x}{2^x - 7^x}$

Soln: We're of the form $\frac{0}{0}$, so using l'H:

$$\lim_{x\to 0} \frac{1-2^x}{2^x-7^x} = \lim_{x\to 0} \frac{\ln 2 \cdot 2^x}{\ln 2 \cdot 2^x - \ln 7 \cdot 7^x}$$

$$= \frac{\ln 2}{\ln 2 - \ln 7}$$

5. $\lim_{x\to 0^-} x^2 \ln x$

Soln: This limit is of the form "$0 \cdot -\infty$" - not quite what we need for l'H. So we rewrite:

$$x^2 \ln x = \frac{\ln x}{\frac{1}{x^2}}$$

$$\lim_{x\to 0^-} x^2 \ln x = \lim_{x\to 0^-} \frac{\ln x}{\left(\frac{1}{x^2}\right)}$$ ← of the form "$\frac{-\infty}{\infty}$", so can use l'H

$$= \lim_{x\to 0^-} \frac{\frac{1}{x}}{-2x^{-3}} = \lim_{x\to 0^-} \frac{\frac{1}{x}}{\frac{-2}{x^3}} = \lim_{x\to 0^-} -\frac{x^3}{2x} = \lim_{x\to 0^-} -\frac{1}{2}x^2$$

$$= 0.$$

6. $\lim_{x\to\infty} x^{-x}$

Soln: This is of the form "$\infty^{-\infty}$"; to use l'H, we'd like to make the exponent into multiplication. To do this, we take the ln of the expression & then exponentiate it:

$$x^{-x} = e^{\ln(x^{-x})} = e^{-x\ln x}$$ so

$$\lim_{x\to\infty} x^{-x} = \lim_{x\to\infty} e^{-x\ln x}$$

$\lim_{x\to\infty} -x\ln x = -\infty$, so $\lim_{x\to\infty} e^{-x\ln x} = 0.$

7. $\lim_{x\to 0^+} x^{-x}$

<u>Soln</u>: We again simplify, since this is of the form "0^0" which is indeterminate.

$$x^{-x} = e^{-x\ln x}$$

$$\lim_{x\to 0^+} x^{-x} = \lim_{x\to 0^+} e^{-x\ln x} = e^{-\lim_{x\to 0^+} x\ln x}$$

(since e^x is cts; as long as this limit exists, ~~this simplifi~~ the last step is valid)

$$\lim_{x\to 0^+} x\ln x \leftarrow \text{"}0\cdot -\infty\text{"}$$

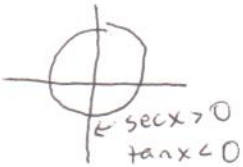
$$= \lim_{x\to 0^+} \frac{\ln x}{\left(\frac{1}{x}\right)} \leftarrow \text{"}\frac{-\infty}{\infty}\text{"}, \text{ so use l'H}$$

$$= \lim_{x\to 0^+} \frac{\frac{1}{x}}{-x^{-2}} = -\lim_{x\to 0^+} \frac{\frac{1}{x}}{\frac{1}{x^2}} = -\lim_{x\to 0^+} x = 0$$

So $e^{-\lim_{x\to 0^+} x\ln x} = e^0 = \underline{1}$.

8. $\lim_{x\to \frac{\pi}{2}^+} \tan x - \sec x$

<u>Soln</u>: This is of the form "$\infty-\infty$", so we would like to rewrite to get a "better" indeterminate form.



$$\tan x - \sec x = \frac{\sin x}{\cos x} - \frac{1}{\cos x} = \frac{\sin x - 1}{\cos x}$$

$$\lim_{x\to \frac{\pi}{2}^+} \frac{\sin x - 1}{\cos x} \leftarrow \text{of the form "}\frac{0}{0}\text{", so can use l'H}$$

$$= \lim_{x\to \frac{\pi}{2}^+} -\frac{\cos x}{\sin x} = 0.$$

9. $\lim_{x\to\infty} x^2 - \ln x$

<u>Soln</u>: We have "$\frac{\infty}{\infty}$", so we simplify first.

$$\lim_{x\to\infty} x^2 - \ln x = \lim_{x\to\infty} x^2\left(1 - \frac{\ln x}{x^2}\right).$$ What is $\lim_{x\to\infty} 1 - \frac{\ln x}{x^2}$?

$$\lim_{x\to\infty} 1 - \frac{\ln x}{x^2} = 1 - \lim_{x\to\infty} \frac{\ln x}{x^2} \leftarrow \text{"}\frac{\infty}{\infty}\text{", so use l'H}$$

$$= 1 - \lim_{x\to\infty} \frac{\frac{1}{x}}{\left(\frac{2x}{1}\right)} = 1 - \lim_{x\to\infty} \frac{1}{2x^2} = 1. \text{ So } \lim_{x\to\infty} \underbrace{x^2}_{\text{grows v. large}}\underbrace{\left(1 - \frac{\ln x}{x^2}\right)}_{\text{goes to } 1.} = \infty.$$