SHORT PROJECT SUGGESTIONS, MATH 191 SPRING 2018

This document will be updated over time. These are intended to be suggestions only – please feel free to search for and/or come up with your own ideas! Many are inspired by other sources.

(1) Investigate knottedness in other dimensions.
  (a) What are slice and ribbon knots? What is the 4-ball genus? How do topologists study these properties of knots?
  (b) Can surfaces be knotted or linked in $\mathbb{R}^4$? Study higher-dimensional knotting in general.

(2) Really understand what knot projections are and why Reidemeister moves suffice to get us between knot projections.

(3) Computational projects:
  (a) Investigate methods for inputting knots into a computer. A good starting point would be Chapter 2 of Adams, which we are not going to talk about in lecture.
  (b) (Also good for long project.) Investigate complexity of recognizing the unknot, e.g. it's in both NP and coNP. What does this mean for P=NP?
  (c) Look into some unknotting algorithms. Try to come up with your own and prove something about its run time in terms of other measurements of a knot's complexity (crossing number is the obvious one, but can you use others?).
  (d) How many crossings do you have to add to a c-crossing projection of the unknot in order to unknot it with Reidemeister moves? Can you bound this number?

(4) Explain p-colorability using linear algebra. Differentiate several knots which we have not differentiated in class using the number of p-colorings for various p.

(5) Investigate physical knots and links in sailing, climbing, etc. See also suggestion 12 in the Spring 2012 short project suggestions.

(6) Learn about the history and mathematical properties of knots and links in art and culture, e.g. Celtic knots, Tibetan Buddhism. Make some conjecture about these knots or explain the proof of a theorem about these knots.

(7) Discuss properties of knots under composition which are and aren't shared with the positive integers under multiplication.

(8) Learn about the knot concordance group and explain an interesting fact or set of examples.

(9) A *Milnor fiber* is the intersection of the zero set of the polynomial $f(u,v) = u^p + v^q$, where u, v are complex numbers, with the three-sphere $\{(u,v) \in \mathbb{C}^2 | |u|^2 + |v|^2 = \epsilon\}$ for some small $\epsilon > 0$. Are these knots familiar? What did Milnor show about these knots? Investigate some other examples – can you find knots whose slice genus equals their genus but aren't obtained in the above way?

(10) See page 113 of Adams for several short and long project ideas about torus knots.

(11) Investigate the knot invariant called *tunnel number*: define it and compute some examples.