

SHORT PROJECT RUBRIC, MATH 191 SPRING 2018

The goal for this paper (beyond learning something yourself!) is to tell a mathematical story. The reader you should have in mind while writing your paper is the other students in the class. Make sure that you prioritize clarity, and everything you say supports the “big picture” story you explain. Don’t just jump right in to calculations – explain why the calculations are relevant.

Each paper should have a significant amount of mathematical work done by you which hasn’t been covered in class. It doesn’t have to be totally original, especially if your paper is a survey or a history of a problem; in those cases, you can provide examples of some of the phenomena described in the theorems you cite. By “significant,” I mean at least half a page to get any credit for this portion of the paper, and more than a full page to get full credit. (All of the project ideas I’ve discussed with you have had plenty of opportunity for this much mathematical work, or more.)

If a theorem you cite is required in a mathematical argument you make, you need to do **one** of the following things:

- (1) If we proved the theorem in class, you can simply reference the date.
- (2) Provide a full proof.
- (3) Provide a reference to a full proof, and a summary of the key ideas.
- (4) In the case where the theorem is truly major (a turning point in the field, with an extraordinarily long proof), you can provide a citation and history/context of the proof instead. The mathematical section of your paper should probably have at most two theorems of this type, depending on your project. If you want advice about this, please ask me.

Take the following example:

Knot composition is analogous to multiplication of natural numbers in several ways. Firstly, just like the natural numbers under multiplication do not form a group, knots under composition do not form a group. Here’s why:

Theorem 1 *Let J and K be knots or links. Then $g(J\#K) = g(J) + g(K)$, where $g(K)$ denotes the genus of K .*

Proof. We proved this in class on Tuesday, February 27. □

Corollary 2 *Knots under composition do not form a group.*

There are more analogies we can make. Knots have unique prime decompositions, just like natural numbers do.

If your paper contained just the above “argument,” then you would not get full credit. You need to include a proof of the corollary, such as

Proof. Note that the genus of the unknot is zero because it has a Seifert surface with genus zero, namely the disk.

Assume that K^{-1} is an inverse for a nontrivial knot K under composition. Then $g(K)$ and $g(K^{-1})$ are both at least one lest they bound a disk, in which case they'd be ambiently isotopic to the unknot (flatten a sub-disk containing the center of the disk, then perform an ambient isotopy which shrinks the boundary radially to the boundary of this flat region).

By Theorem 1, $g(U) = g(K\#K^{-1})$. Therefore,

$$0 = g(U) = g(K\#K^{-1}) = g(K) + g(K^{-1}) \geq 1 + 1$$

which implies $0 > 1$, a contradiction. □

Note it's fine to cite and give background for a major theorem, without proving it or summarizing its proof, if it appears in a survey and/or historical section of your paper in which you are comparing and contrasting different results, rather than in a mathematical argument you are making. For example, if your project is on Vassiliev invariants, you might write something like:

In 1833, Gauss introduced the linking number by means of an integral (you'd provide a citation here!). In 1993, Kontsevich generalized Gauss's linking integral in order to understand the Vassiliev invariants of knots (you'd cite Kontsevich's paper here!).

Note that the above example does not count as mathematical content, however, since you don't prove or compute anything.

Formatting guidelines were passed out previously, and are linked to my website. For your bibliography, you can use bibtex, or LaTeX's internal bibliography tools. When you cite an article, provide the author(s), title, journal (with issue/volume information), pages, and year.

Grading will be as follows:

Mathematics: Do you have at least half a page of mathematical content new to the course? Is it correct? Do you provide sufficient definitions for and explanation of new ideas, and are they presented in logical order?	10 points
Exposition: Does the paper communicate your ideas effectively to your fellow 191 students? Is the paper globally arranged in logical order? Do you support your ideas and explain all your claims? Is your writing concise and direct?	10 points
Presentation: Is the formatting correct? Is the paper free of obvious spelling and/or typesetting errors? Is the paper relatively easy to read, e.g., you use complete English sentences except during computations, and you explain all steps of proofs and computations? Are all sources cited properly?	6 points
15 minute meeting: did we have one?	1 point
Progress report: did you turn one in?	1 point
Discussion day: Did you attend and participate? When I listened in on a small group you were part of, were you both actively listening and communicating clearly about your project?	2 points