§7.4 Variance. Some useful definitions and facts:

- The **variance** of a random variable X on a sample space S is defined by
$$V(X) = \sum_{s \in S} (X(s) - E(X))^2 p(s)$$
and the **standard deviation** is defined by $\sigma(X) = \sqrt{V(X)}$.
- Notice that $V(X) = E((X - E(X))^2)$.
- We can show straight from the definition that $V(X) = E(X^2) - E(X)^2$.
- An **indicator random variable** for an event E is defined by
$$X(s) = \begin{cases} 1 & \text{if } s \in E \\ 0 & \text{if } s \notin E \end{cases}$$
- For pairwise independent random variables $X_1, \ldots, X_n$ on a sample space s,
$$V(X_1 + \cdots + X_n) = V(X_1) + \cdots + V(X_n)$$
- **Chebyshev's inequality** says that for X a random variable on a sample space S with probability function p and r a positive real number,
$$p(|X(s) - E(X)| \geq r) \leq \frac{V(X)}{r^2}$$
and we can show this straight from the definition of variance.

Some problems:

(1) Suppose a fair octahedral die (eight faces) and a fair dodecahedral die (twelve faces) are rolled. What is the expected value of the sum of the numbers that come up? What is the variance of the sum of the numbers that come up?

(2) Let X be a random variable on a sample space S. Show that $V(aX + b) = a^2 V(X)$ whenever a and b are real numbers.

(3) Let X_n be the random variable that equals the number of tails minus the number of heads when n fair coins are flipped. What is the expected value of X_n? What is the variance of X_n?

(4) Let X_1 be the number that comes up when rolling one fair die and Y be the sum that comes up when rolling two fair dice. Check whether or not $V(X_1 + Y) = V(X_1) + V(Y)$.

(5) Use Chebychev's inequality to find an upper bound on the probability that the number of tails that come up when a biased coin with probability of heads equal to 0.6 is tossed n times deviates from the mean by more than $\sqrt{n}$.