Introduce yourselves, then choose a scribe for the first problem who will write your group's solutions on the board. When you're done with each problem, switch scribes. Feel free to ask for help whenever you need!

§2.5 Cardinality.

(1) Show that $|[0,1)| = |\mathbb{R}|$.

(2) Show that the union of countably many countable sets is countable. Hint: don't try to write down an explicit bijection; instead show some way to enumerate all the elements in the union. Recall the proof of the countability of $\mathbb{Q}$.

(3) (a) Prove that the set of grammatically sensible English language sentences is countable.
(b) Use part (a) to show that there is a real number that cannot be described in English.
(c) How is this question related to the existence of uncomputable functions? Hint: there are uncountably many functions from $\mathbb{Z}^+$ to $\{0, 1, \ldots, 9\}$, as we can see by associating to the real number $0.d_1d_2 \cdots d_n \cdots$ the function $f(n) = d_n$.

(4) (a) Show that the set of all real numbers that are the roots of some polynomial with integer coefficients is countable.
(b) Show that the set of all real numbers that are the roots of some polynomial with rational coefficients is countable.
(c) Use part (b) to show that there are uncountably many real numbers that are not the root of any polynomial with rational coefficients. Such real numbers are called "transcendental." Can you think of any examples?