

Introduce yourselves, then choose a scribe for the first problem who will write your group's solutions on the board. When you're done with each problem, switch scribes. Feel free to ask for help whenever you need!

### §2.3 Functions.

- (1) Determine whether each of the following are injections, surjections, bijections, or none of the above:
  - (a)  $f : \mathbb{R} \rightarrow \mathbb{R}, f(x) = 3x - 2$
  - (b)  $f : \mathbb{R} \rightarrow \mathbb{Z}, f(x) = \lfloor x \rfloor$
  - (c)  $f : \mathbb{N} \rightarrow \mathbb{N}, f(n) = n + 1$
  - (d)  $f : \mathbb{R} \rightarrow \mathbb{R}, f(x) = x(x - 3)(x + 2)$
- (2) True or false? If true, prove it. If false, find a counterexample.
  - (a) If  $f$  and  $g$  are injective, then so is  $f \circ g$ .
  - (b) If  $f$  and  $g$  are surjective, then so is  $f \circ g$ .
  - (c) If  $g \circ f$  is injective, then so is  $f$ .
  - (d) If  $g \circ f$  is injective, then so is  $g$ .
  - (e) If  $f$  is not surjective, then  $g \circ f$  is not surjective.
- (3) Explain why non-injective functions have no inverse.
- (4) Give an example of an injective function from  $\mathbb{N}$  to  $\mathbb{N}$  which is not surjective.

### §2.4 Sequences and Summations.

- (1) Show that the sequence  $\{a_n\}$  is a solution of the recurrence relation  $a_n = -3a_{n-1} + 4a_{n-2}$  if
  - (a)  $a_n = 0$
  - (b)  $a_n = 1$
  - (c)  $a_n = (-4)^n$
  - (d)  $a_n = 2(-4)^n + 3$
- (2) Compute each of the sums
  - (a)  $\sum_{j=0}^8 (3^j - 2^j)$
  - (b)  $\sum_{i=1}^3 \sum_{j=0}^2 i^2 j^3$
- (3) Show that  $\sum_{j=1}^n (a_j - a_{j-1}) = a_n - a_0$ , where  $a_0, a_1, \dots, a_n$  is a sequence of real numbers.