

Introduce yourselves, then choose a scribe for the first problem who will write your group's solutions on the board. When you're done with each problem, switch scribes. Feel free to ask for help whenever you need!

### §1.1-1.6 Review.

- (1) (a) Find a compound proposition that has the following truth table:

p	q	r	???
T	T	T	T
T	T	F	F
T	F	T	F
T	F	F	T
F	T	T	F
F	T	F	T
F	F	T	F
F	F	F	F

- (b) How could you generalize your method to any number of variables and any truth table? This is what it means that  $\neg, \wedge, \vee$  are a **complete** set of connectives.

- (2) If  $P(x)$  is the statement " $x > 0$ " and  $G(x, y)$  is the statement " $x^2 \geq y$ ," determine the truth values of each of the following statements when the domain is all of the real numbers:
- $\forall x P(x)$
  - $\exists x P(x) \wedge \forall x G(x, 0)$
  - $\exists y G(2, y)$
  - $\neg P(x) \rightarrow \neg G(x, x)$
- (3) Determine what conclusions you can draw from the following sets of premises:
- "All athletes watch professional sports," "Alice is an athlete."
  - "All athletes watch professional sports," "Bob watches professional sports."
  - "All athletes watch professional sports," "Everyone who watches sports likes pizza," "Charlie doesn't like pizza."
  - "All athletes watch professional sports," "Dan isn't an athlete."

### §1.7 Introduction to Proofs and §1.8 Proof Methods and Strategy.

- Prove that if  $3n + 2$  is odd, then  $n$  is odd.
- Prove that if  $n$  is a positive integer, then  $n$  is even if and only if  $7n + 4$  is even.
- Prove that the sum of a rational number and an irrational number is irrational.
- Prove that if  $a, b, n$  are positive integers such that  $ab > n$ , then  $a > \sqrt{n}$  or  $b > \sqrt{n}$ .

- (5) Challenge: show that there are two irrational numbers  $x$  and  $y$  such that  $x^y$  is rational. Hint: if  $\sqrt{2}^{\sqrt{2}}$  is irrational, what can we set  $x$  and  $y$  to be to get  $x^y$  rational? (If  $\sqrt{2}^{\sqrt{2}}$  is rational, then we're done: why?)

### §2.1 Sets and §2.2 Set Operations.

- (1) How many elements are in each of the following sets?
- (a)  $\{1, 2, 3\} \cup \{1, 4, \emptyset\}$
  - (b)  $\mathcal{P}(\{a, b, c, d\} - \{a, e\})$
  - (c)  $A \cup \{A\}$  where  $|A| = n$
- (2) If  $|A| = 5$ ,  $|B| = 3$ , and  $|A \cap B| = 1$ , what is  $|A \cup B|$ ?
- (3) Prove that for any sets  $A, B, C$ ,  $(A - B) \cup C \subseteq A \cup C$ .
- (4) Prove that  $A \subseteq \overline{B - A}$