Chain rule for $\frac{d^2y}{dx^2}$

Think of y as a function $f(x)$ of x and x as a function $g(t)$ of t. Then $\frac{dy}{dx} = f'(x) = f'(g(t))$, which is a function of t. Therefore, by the chain rule,

$$\frac{d}{dt}\frac{dy}{dx} = \frac{d}{dt}f'(g(t)) = f''(g(t))g'(t) = f''(x)g'(t) = \frac{d^2y}{dx^2}\frac{dx}{dt}$$

and we can rearrange to get the formula for $\frac{d^2y}{dx^2}$ that we want, by dividing both sides by $\frac{dx}{dt}$. The tricky part is that you have to keep in mind that $f'(x)$ is a function of t, and the trick is to take $\frac{d}{dt}$ of $f'(x)$.

Worksheet 1 Problem 3(c)

The graph of $x_1 = t^2 + \frac{1}{t^2}$, $y_1 = t^2 - \frac{1}{t^2}$ is a subset of the graph of $x_2 = t + \frac{1}{t}$, $y_2 = t - \frac{1}{t}$ (by "subset" I mean that the picture of the first graph is entirely contained in the picture of the second graph). This is because any point on the first graph can be replicated on the second graph by plugging in t^2: plug in 5 to x_1, y_1 and plug in 25 to x_2, y_2, and you'll get the same point.

But it is not the *entire* second graph: since t^2 can never be negative, I can never get any point as (x_2, y_2) which requires me to plug in a negative value of t in (x_1, y_1). That is, the point $\left(-25 + \frac{1}{-25}, -25 - \frac{1}{-25}\right)$ will never appear as a point in the graph of (x_1, y_1), but it does appear in the graph of (x_2, y_2).

In part **(b)** of this problem, you showed that if $t < 0$, $x_2 < 0$, and if $t >$¡ $x_2 > 0$ (you can't plug in $t = 0$ to either set of equations). So the part of the graph if x_2, y_2 which I have to "cut out" to get the graph of x_1, y_1 is the part where $x < 0$.

The question now remains: do I get *all* of the graph of x_2, y_2 where $x > 0$, i.e. the whole right branch of the hyperbola? Certainly I can replicate every value of x_2 as a value of x_1 (just plug in the square root), but can I also get all the y values?

Another way to ask this question is this. Say $(x_2, y_2) = \left(t + \frac{1}{t}, t - \frac{1}{t}\right)$ is a point on the graph of x_2, y_2, where $t > 0$. Can I find some s to plug in to x_1, y_1 to get $\left(s^2 + \frac{1}{s^2}, s^2 - \frac{1}{s^2}\right) = \left(t + \frac{1}{t}, t - \frac{1}{t}\right)$? If I try $\sqrt{t}$, which exists because $t > 0$, I should get the same point.

Another thing to notice is that x_1, y_1 will cover the right branch of the hyperbola twice, because plugging in t and $-t$ will give you the same point (this is because $t^2 = (-t)^2$).

In discussion I may have said something about checking $|t| < 1$ and $|t| > 1$; this isn't necessary, because you can directly take the square root of any positive number.