Morgan Weiler: Research Statement

Symplectic and contact dynamics, embedded contact homology, and spectral invariants

1 Introduction

My research is in low-dimensional symplectic and contact geometry, focusing on dynamics in two and three dimensions. In Theorem 1 ([38, Theorem 1.9]) I show[1] that many area-preserving annulus diffeomorphisms have *periodic points* (points fixed under some iterate of the diffeomorphism), and I provide an upper bound for how far these diffeomorphisms distort curves near their periodic points.

The results in [38] arise from classical questions in symplectic geometry. A *symplectic form* is a closed, nondegenerate two-form, which in two dimensions is an area form. The foundational example is phase space, the even-dimensional Euclidean space whose coordinates record the position and momentum of a moving particle. Smooth functions on phase space generalize the concept of energy, so in order to understand motion which respects the conservation of energy, we restrict attention to the level sets of smooth functions. The symplectic form determines a preferred vector field whose flow preserves the level sets of a given function and which represents the possible tangencies to the motion of a particle. Contact dynamics is the dynamics of "Reeb vector fields," an abstraction of these vector fields to other manifolds [1] (terms in quotations defined in §2).

In his work on celestial mechanics, Poincaré noticed that the dynamics of a Reeb vector field can be studied using the dynamics of diffeomorphisms on a surface transverse to its flow. This led him to conjecture in [34] that if an area-preserving diffeomorphism of an annulus with boundary twists the boundary components in opposite directions, then it has at least two fixed points. Birkhoff proved Poincaré's conjecture in 1925 [3]. In 1992, Franks proved that an area-preserving homeomorphism of an annulus has either zero or infinitely many periodic points [13]. The area-preserving assumption is crucial (see [29, Chapter 8]), and generalizes to contact and symplectic geometry: see [17, 14, 8] for a similar dichotomy for orbits of Reeb vector fields and other symplectic vector fields.[2]

My work takes a quantitative, intrinsically symplectic perspective on the dynamics of area-preserving surface diffeomorphisms. In [38] I combine Floer-theoretical spectral invariants with "open book decompositions," a global version of Poincaré's transverse surfaces. One spectral invariant is a "knot filtration" on "embedded contact homology" (ECH). ECH is a powerful invariant in three-dimensional dynamics, and was used to prove the longstanding Weinstein and Arnold chord conjectures in dimension three [37, 25]. In [38] I generalized the filtration from [20]; in [38] and work in progress I have computed many examples of knot-filtered ECH. In future research, I will develop the foundations of knot-filtered ECH and apply it to dynamics and contact topology, in particular extending the the results from [38] to a quantitative characterization of surface dynamics. I will also create new computational approaches for J-holomorphic curve invariants, including symplectic embedding obstructions, and investigate connections with Heegaard Floer and contact homologies.

2 Previous work

2.1 Main result

Denote by (A, ω) the annulus $[-1, 1]_x \times (\mathbb{R}/2\pi\mathbb{Z})_y$ with area form $\omega = \frac{1}{2\pi}\, dx \wedge dy$. Let $\beta = \frac{x}{2\pi}\, dy$; note that β is a primitive for ω. Let ψ be an area-preserving diffeomorphism of (A, ω) which is a rotation by $2\pi y_\pm$ near ∂A (note the choice of $y_+ \in \mathbb{R}/\mathbb{Z}$ determines y_-). The *action function* of (ψ, y_+) is the function $f : A \to \mathbb{R}$ with $df = \psi^*\beta - \beta$ and $f(1, y) = y_+$. It is constant near ∂A. If (x, y) is a fixed point of ψ, let η be a curve from (x, y) to $\{x = 1\}$. Then $f(x, y)$ is an integer

[1]Partially supported by the NSF grant EMSW21-RTG-1344991.

[2]The same cannot be said of volume-preserving geometry (see [26]), so "symplectic/contact" is the correct generalization of "area-preserving" which extends this delicate dynamical balance to higher dimensions.

(depending on y_+) plus the area between η, $\psi(\eta)$, and $\{x = 1\}$ (Fig. 1). The *Calabi invariant* of (ψ, y_+) is the number

$$\mathcal{V}(\psi, y_+) := \frac{\int_A f\omega}{\int_A \omega}$$

A *periodic orbit* γ of ψ of *period* $\ell(\gamma)$ is the list of iterates of γ_1 under ψ, where $\psi^{\ell(\gamma)}(\gamma_1) = \gamma_1$. The *total action* of γ is $\mathcal{A}(\gamma) := \sum_{i=0}^{\ell(\gamma)-1} f(\psi^i(\gamma_1))$. The ratio $\frac{\mathcal{A}(\gamma)}{\ell(\gamma)}$ is the *mean action* of γ.

Let $\mathcal{P}(\psi)$ denote the set of periodic orbits of ψ. My first result is:

Theorem 1 (Weiler, [38, Theorem 1.9])**.** *Let y_+, $y_- \in \mathbb{R}$. Let ψ be an area-preserving diffeomorphism of (A, ω) which is rotation by $2\pi y_\pm$ near the $x = \pm 1$ boundary, respectively. Assuming*

$$\mathcal{V}(\psi, y_+) < \max\{f(1, y), f(-1, y)\} \tag{1}$$

then

$$\inf\left\{\frac{\mathcal{A}(\gamma)}{\ell(\gamma)} \;\middle|\; \gamma \in \mathcal{P}(\psi)\right\} \leq \mathcal{V}(\psi, y_+) \tag{2}$$

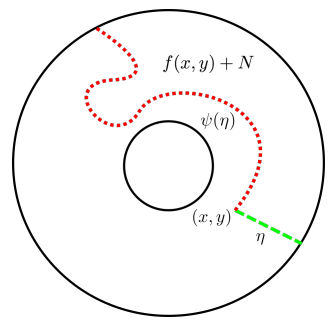

Figure 1: *Action at a fixed point, up to $N \in \mathbb{Z}$.*

Theorem 1 detects periodic points in many cases not covered by [3, 13]. It also provides a quantitative illustration of Franks' all-or-nothing dichotomy. The hypothesis (1), together with the corresponding hypothesis of the corollary obtained by replacing ψ with ψ^{-1}, show that unless $\mathcal{V}(\psi, y_+) = f(1, y) = f(-1, y)$, the map ψ has periodic points. That is, the condition of having zero periodic points is both quantitatively and qualitatively delicate. Moreover, Theorem 1 tells us about the geometry of ψ. Any function attains a value less than or equal to its average at some point, but (2) shows there is a "below average"[3] point z for the action function for which $\psi^\ell(z) = z$ for some ℓ. Then the bound (2) constrains how ψ^ℓ distorts curves near z, as in Fig. 1.

2.2 Tools from contact geometry

We next explain Proposition 2, which describes the connection between two- and three-dimensional dynamics. Let Y denote a closed oriented three-manifold. A *contact form* λ on Y is a one-form with $\lambda \wedge d\lambda > 0$. Its *contact structure* is the $\mathbb{R}^2$ bundle $\xi = \ker \lambda$ and its *Reeb vector field* is the smooth vector field R on Y defined by $d\lambda(R, \cdot) = 0$ and $\lambda(R) = 1$. A *Reeb orbit* is a smooth map $\gamma : \mathbb{R}/T\mathbb{Z} \to Y$, modulo reparameterization, with $\dot\gamma(s) = R(\gamma(s))$. It has *symplectic action* $\mathcal{A}(\gamma) = \int_\gamma \lambda = T$. A Reeb orbit γ is *elliptic* if the derivative of the Reeb flow, restricted to the contact planes, is conjugate to a rotation. In this case it has a *rotation number* $\mathrm{rot}(\gamma)$, which is the argument of this rotation divided by 2π, and which depends on a framing of $\gamma^*\xi$.

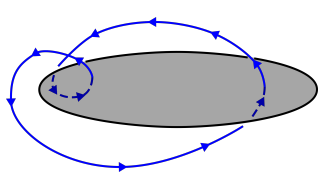

Figure 2: *An orbit transverse to a disk.*

Given an oriented link B in Y, an *open book decomposition* of Y with *binding* B is a fibration $\Pi : Y - B \to S^1$ where $\Pi^{-1}(\theta)$ are interiors of Seifert surfaces for B. A contact form with Reeb vector field R is *adapted* to (B, Π) if R is positively tangent to B, positively transverse to the pages of Π, and each point in $Y - B$ takes finite time under the flow of R to return to its original page. The *return time* is the function from the page to $\mathbb{R}$ recording this time. The *return map* is a diffeomorphism of the page, defined for $z \in Y - B$ as the point in $\Pi^{-1}(\Pi(z))$ where the flow of R sends z at its return time. If λ and λ' are both adapted to (B, Π), they have *contactomorphic* contact structures (there is a diffeomorphism ϕ of Y with $\phi_* \ker \lambda = \ker \lambda'$): see [16, Proposition 2], [10, Theorem 1.1].

[3] plus any small $\epsilon > 0$, since we are using infimum rather than minimum

Proposition 2 (Weiler, [38, Proposition 3.1])**.** *For (ψ, y_+) as in the hypotheses of Theorem 1 along with further technical assumptions,*[4] *set $\tilde{p} = f(1,y) + f(-1,y)$.*

There is a contact form $\lambda_{\tilde{\psi}}$ on the lens space $L(\tilde{p}, \tilde{p}-1)$, adapted to an open book decomposition with annulus pages, with return time f and return map ψ. Its elliptic binding orbits have rotation numbers $f(1,y)^{-1}$ and $f(-1,y)^{-1}$. There is a bijection from the other orbits to $\mathcal{P}(\psi)$, sending symplectic action to total action, and number of intersections with a single page to period.

Part of the proof of Theorem 1 is a lower bound on the periods of orbits of ψ. We obtain it by computing the ECH of $(L(\tilde{p}, \tilde{p}-1), \lambda_{\tilde{\psi}})$ filtered by the linking number of Reeb orbits with the binding components of the open book decomposition from Proposition 2. ECH is a Floer homology theory whose chain groups are generated[5] over $\mathbb{Z}/2\mathbb{Z}$ by certain finite sets α of embedded Reeb orbits. Given generators α_0 and α_1, the coefficient of α_1 in the differential of α_0 is a count of J-holomorphic curves[6] approaching α_0 in $\{s\} \times Y$ as $s \to \infty$ and α_1 in $\{s\} \times Y$ as $s \to -\infty$. If $c_1(\ker \lambda)$ is torison, ECH is $\mathbb{Z}$-graded by the *ECH index*. A *transverse knot* B in Y with contact structure ξ is a knot transverse to $\xi \subset TY$. For an ECH generator α of (Y, λ), let $\mathcal{F}_B(\alpha)$ be the total linking number of α with B, plus a correction term if B is in α. Denote by $ECH_*^{\mathcal{F}_B \leq \ell}(Y, \lambda)$ the homology of the ECH chain complex generated by α_0 with $\mathcal{F}_B(\alpha_0) \leq \ell$.

It is not possible to compute $ECH_*^{\mathcal{F}_B \leq \ell}(L(\tilde{p}, \tilde{p}-1), \lambda_{\tilde{\psi}})$ directly. By [16, Proposition 2], [10, Theorem 1.1], and the following theorem, the filtered ECH of any contact form adapted to the open book decomposition from Proposition 2 is isomorphic to the filtered ECH of $\lambda_{\tilde{\psi}}$.

Theorem 3 (Weiler, [38, Theorem 7.1])**.** *The homology $ECH_*^{\mathcal{F}_B \leq \ell}(Y, \lambda)$ depends only on Y, the contact structure* $\ker \lambda$*, the transverse knot B, and its rotation number rot(B).*

For a, $b \in \mathbb{R}$, let $N_k(a,b)$ denote the kth term in the sequence of nonnegative integer linear combinations of a and b, listed with repetition, starting with $N_0(a,b) = 0$. The proof of the following result uses methods similar to those in the proof of [38, Proposition 5.7].

Proposition 4 (Weiler)**.** *Given ψ as above, let $e_\pm$ denote the binding orbits of the open book decomposition from Proposition 2. There are functions $w_\pm(k) \geq k$ for which*

$$ECH_{2k}^{\mathcal{F}_{e_\pm} \leq \ell}(L(\tilde{p}, \tilde{p}-1), \lambda_{\tilde{\psi}}) = \begin{cases} \mathbb{Z}/2\mathbb{Z} & \text{if } \ell \geq N_{w_\pm(k)}\left(1, \frac{1}{f(\pm 1, y)}\right) \\ 0 & \text{else} \end{cases}$$

The sum of the linking numbers with the binding equals the intersection number with the page, so [38, Proposition 5.7] allows us to obtain the lower bound on the periods of orbits of ψ. As discussed in §3.2, Proposition 4 provides hints towards the structure of knot-filtered ECH.

3 Future research objectives

3.1 Refined dynamics on the annulus

The mean action of any cover of an orbit γ of ψ equals the mean action of the underlying orbit γ, so the distribution of the mean action values of ψ reflects the qualitative dynamics of ψ. I will work towards a quantitative characterization of the Franks dichotomy.

Problem 5. Improve our understanding of the distribution of mean action values beyond (2), starting on disks, annuli, and other genus zero surfaces. Specifically:

1. For what ψ does the strict upper bound $\inf \left\{ \frac{\mathcal{A}(\gamma)}{\ell(\gamma)} \,\middle|\, \gamma \in \mathcal{P}(\psi) \right\} < \mathcal{V}(\psi, y_+)$ hold?

[4] See [38] for these assumptions. Most importantly, we require $f > 0$.

[5] The coefficients can be strengthened to $\mathbb{Z}$, see [19] and references therein for details.

[6] These are solutions to the Cauchy Riemann equations on $\mathbb{R} \times Y$ for a generic complex structure J on $T(\mathbb{R} \times Y)$.

2. What ψ have an infinite sequence of mean action values (whether less than the Calabi invariant or not)? When is the set of mean action values is somewhere dense?

3. In a fixed period, what is the largest gap between total action values? Can it be expressed in terms of the area of the annulus, or the flux or Calabi invariant of the diffeomorphism?

4. If (2) does not hold when $\epsilon = 0$, what does that tell us about ψ?

5. If (2) is an equality, what does that tell us about ψ?

One possible set of conditions sufficient for 1. are the hypotheses of [15, Theorem 3.1], in which Ginzburg-Gürel strictly separate a sequence of (a filtered Floer homology analogue of) mean action values from the quantity $\hat{c}_\infty$, which in some cases is an extension of the Calabi invariant (see [9]). However, 1. would not follow directly from an ECH version of [15, Theorem 3.1], because it provides a lower rather than upper bound.

Alternately, 1. may follow from the following approach to 2. I plan to extend Theorem 1 to more genus zero surfaces (see Problem 12 for the beginning of this project). Given a diffeomorphism ψ satisfying the hypotheses of Theorem 1, we we can modify ψ near a point in the orbit from Theorem 1, producing a new diffeomorphism on a surface with one more boundary component. If an analogue of Theorem 1 holds for this new diffeomorphism, we get another orbit. Iterating this procedure would produce infinitely many orbits; with care, it could produce a decreasing sequence of infinitely many mean action values, all less than the original Calabi invariant.

It may also be possible to use 3. to address 2. I will approach 3. via the bifiltration on ECH by both symplectic action and linking number. 4. and 5. will require different techniques, perhaps from low-dimensional dynamics or a direct application of J-holomorphic curves, e.g. [4, 5, 28]. 5. could follow from methods similar to those of Hryniewicz-Momin-Saomão in [18].

3.2 Work in progress: positive torus knots

There is one computation of knot-filtered ECH besides those in [20, 38]. Denote the (p, q) torus knot for $pq > 0$ by $T_{p,q}$. There are contact forms $\lambda^{\pm}_{p,q}$ on S^3 with $T_{p,q}$ as a Reeb orbit, with $\mathrm{rot}(T_{p,q}) = pq \pm \epsilon$ for small ϵ in a standard framing, and $\ker \lambda^{\pm}_{p,q}$ contactomorphic to a standard contact structure. Let $n_k(a, b, c)$ be the kth term in the sequence of nonnegative integer linear combinations of a, b, and c, listed without repetition, starting with zero. Work in progress suggests:

Conjecture 6.

$$ECH_{2k}^{\mathcal{F}_{T_{p,q}} \leq \ell}(S^3, \lambda^{\pm}_{p,q}) = \begin{cases} \mathbb{Z}/2\mathbb{Z} & \textit{if } \ell \geq n_k(p, q, pq \pm \epsilon) \\ 0 & \textit{else} \end{cases}$$

A *toric* contact form is one with a Reeb vector field tangent to the integral surfaces of a T^2 action. Because $n_k(p, q, pq \pm \epsilon)$ converges to $N_k(p, q)$ as $\epsilon \to 0$, and the contact forms used in [20], Proposition 4, and Conjecture 6 are either toric or small perturbations of toric forms, these results suggest the toric condition has significant structural implications for knot-filtered ECH.

3.3 Computations of knot-filtered ECH

The main challenge in extending Theorem 1 to other surfaces is computing the knot-filtered ECH of the three-manifold obtained in the analogue of Proposition 2. The relatively simple characterizations above in terms of the N_k and n_k sequences are not likely to hold in general. However, direct computations of Floer homologies are very difficult without strong topological restrictions such as the toric condition. The following problems address computations which will be tractable, illuminating, and significantly extend current results.

One approach is to leverage any algebraic structure corresponding to natural geometric operations on transverse knots which are bindings of open book decompositions. The *Murasugi sum* of two open book decompositions on Y and Y' is an open book decomposition on $Y\#Y'$ whose pages are obtained by plumbing the pages of the components (thus connected summing their bindings).

Problem 7. Can the knot-filtered ECH of the binding of a Murasugi sum be expressed algebraically in terms of the knot-filtered ECH of the bindings of its components?

Next, computations so far show that knot-filtered ECH depends heavily on rotation number, but we do not know how much.

Problem 8. Compute $ECH_*^{\mathcal{F}_{T_{p,q}} \leq \ell}(S^3, \lambda)$ for $\ker \lambda$ contactomorphic to $\ker \lambda_{std}$ and $|\mathrm{rot}(T_{p,q}) - pq| > 1$, with the same transverse type[7] (of $T_{p,q}$) and framing (for $\mathrm{rot}(T_{p,q})$) as in Conjecture 6.

Any work on Problem 8 will require new methods, since the contact forms are not close to toric. We would also like to know how knot-filtered ECH depends on the transverse type of the knot.

Problem 9. How does knot-filtered ECH depend on self-linking number[8]?

I expect to be able to compute the knot-filtered ECH in at least one transverse type of some iterated torus knots using their structure as the link of a complex singularity. An interesting example is the $(2,3)$-cable of $T_{2,3}$. Etnyre-Honda showed it has transverse types not distinguished by classical invariants [11], making it a good candidate for computations of knot-filtered ECH.

Problem 10. Compute the knot-filtered ECH of the $(2,3)$-cable of $T_{2,3}$ in more than one of its transverse types. Does knot-filtered ECH distinguish the non-classical transverse types from [11]?

To explore dependence on the contact structure, I will use contact forms adapted to simple open book decompositions. This is promising because open book decompositions are a strong computational aid in Heegaard Floer homology ([32, 2]), which is isomorphic to ECH ([6, 27]).

Problem 11. The negative trefoil and figure-eight can be realized as the binding of genus one open book decompositions on S^3 adapted to nonstandard contact forms. Compute their knot-filtered ECH. Extend these techniques to prove a version of Theorem 1 for the torus minus an open disk.

Problem 12. Circle bundles on surfaces admit relatively simple open book decompositions, adapted to contact forms whose Reeb vector field is parallel to the S^1 action. Compute the knot-filtered ECH for any binding component of these open book decompositions of circle bundles on surfaces, extending [12].

For Problems 11 and 12, I will extend the methods from [24, 12] in ECH and [7, 30] in contact homology. I will construct a model contact form adapted to the open book decomposition, then simplify the dynamics of its Reeb vector field using a geometric perturbation of the contact form. Here, "geometric" indicates that the perturbation is parameterized by a page. These perturbations are very explicit in the cases of the trefoil and circle bundles; the case of the figure-eight knot must be handled differently, combining techniques from [23] in periodic Floer homology and [7] in contact homology.

[7] i.e., up to isotopy through transverse knots

[8] The *self-linking number* of a transverse knot is its linking number with a pushoff specified by a framing of ξ over the knot which extends to a trivialization of ξ over a Seifert surface for the knot. It is the main classical transverse knot invariant.

3.4 Knot-filtered ECH and Heegaard Floer homology

In [33], Oszváth-Stipsicz-Szabó introduce a knot invariant, Υ_K, constructed from a filtration on knot Floer homology similar to our knot filtration, and use Υ_K to prove bounds on slice genus.

Problem 13. 1. Construct a Υ-type invariant in ECH. Is it a smooth or contact-geometric invariant? Do its properties cause knot-filtered ECH to be a smooth or contact invariant?

2. If the ECH "Υ invariant" is a contact-geometric invariant, what are its applications? (E.g. bounds on the genus of fillings of a knot in a symplectic filling[9] of its contact manifold.)

A key ingredient in the construction of Υ_K is a degree -2 map on Heegaard Floer homology called the U map. There is an analogous U map in ECH, and any work on Problem 13 would require a deep understanding of this map. Moreover, the U map in ECH is the key to proving major results in Reeb dynamics, in particular [8] in which Cristofaro-Gardiner-Hutchings-Pomerleano prove that many Reeb vector fields have two or infinitely many simple orbits. However, it has only been directly computed for some toric contact forms.

Problem 14. Compute the U map for contact forms whose Reeb vector field traces out a fibration by S^1, and on Seifert fibered spaces. Can we compute this map in more general cases, for example, for contact forms adapted to low genus open book decompositions?

3.5 Computational approach for the ECH spectrum of S^1-invariant contact forms

I will investigate computations of the filtration on ECH by symplectic action for contact forms invariant under an S^1 action. This filtration is the key to the determining the *ECH spectrum*, one of the most widely used obstructions to symplectic embeddings, and S^1-invariant contact forms are a natural extension of my other research interests. The most straightforward generalization from forms whose Reeb vector field is tangent to an S^1 fibration is the class of forms whose Reeb orbits trace out a Seifert fibered space; these arise as contact forms adapted to open book decompositions for which some iterate of the return map is the identity [7], which I will address in Problems 11 and 12. The next generalization is to contact forms which are only invariant under an S^1 action.

Problem 15. Can we compute the action filtration on the ECH of S^3 in the standard contact structure, using a contact form which is invariant under an S^1 action but whose Reeb vector field does not trace out an S^1 fibration? (We have explicit examples of such forms.)

Work on Problem 15 will begin to generalize the results of Ramos and Ramos-Sepe using billiards to study embeddings of Lagrangian products in [35, 36]. Any approach to Problems 14 and 15 will require a deep understanding of the analysis of J-holomorphic curves in symplectizations. I will extend the methods of [12, 24] in ECH, [23] in periodic Floer homology, as well as [31, 22, 21] in contact homology. My methods will rely on geometric perturbations of the S^1 actions, parameterized in this case by the quotient of the contact manifold by the action.

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[9] A *symplectic filling* is a smooth filling with a symplectic form which is sufficiently compatible with the contact form on its boundary.

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